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MRI Image Denoising Approach based on TV and Neural Network Filter

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***Abstract*:** To reduce the Gaussian noise from Magnetic Resonance Image (MRI) corrupted during their acquisition process, we propose a filtering method based RBF neural network. Indeed, the Gaussian noise is considered and formulated as constraints in an energy functional base on minimization of Total Variation (TV). In the RBF training stage, the backprobagation algorithm is used to solve the TV functional energy, where the reaches image is its solution. The considered filter has given good results of noise removal when compared to other approaches.

Keywords: Noise reduction; MRI Images; Total variation; RBF neural network.

1. Introduction

Magnetic Resonance Image (MRI) is an imaging method commonly used in medical setting to generate high quality images and provide more effective information of the inside of the human body. MRI images are always corrupted with noise. Removing noise from images is a crucial step to increase image quality and to improve the performance of all the tasks needed for quantitative imaging analysis.

Numerous approaches of denoising MR images have been proposed in the literatures. Henkelman [1] was the first to estimate the magnitude MR image from a noisy image. PDE based methods, especially anisotropic diffusion proposed by Gerig et al. [2] are able to remove noise from MRI using gradient information while respecting important image structures.

A variant of standard anisotropic diffusion method was extended by Yang et al. [3] using both the local intensity orientation and an anisotropic measure of level counters, instead of using local gradients to control the anisotropism of the filters. Using anisotropic diffusion or its variants can generate a bias in the magnitude MR data (which increases with decreasing SNR). Sijbers et al [4] proposed an adaptive anisotropic diffusion method to tackle this problem. Krissian and Aja Fernandez [5] proposed a new anisotropic diffusion filter based on a linear minimum mean squared error estimation and partial difference equations for Rician noise removal.

A simple wavelet based noise reduction was proposed by Weaver et al. [6]. The main drawback of the method was that small structures similar in size to noise where also eliminated. Wavelet transform based denoising for Rician noise removal in MRI was proposed by Nowak [7], where the magnitude of the MR image was squared and the square of a Rician random variable was modeled by a scaled noncentral chi-square distribution. A wavelet denoising method was compared with Gaussian smoothing methods by Wink and Roerdink [8]. A Wiener like-filtering method was applied by Wirestam et al [9] in the wavelet domain before the reconstruction of MR images.

Yang and Fei [10] proposed the wavelet multiscale denoising method for the Rician nature of MR data. In this method, Radon transform is applied to the original MR images and the Gaussian noise model is used to process the MR sinogram image. A translation invariant wavelet transform is employed to decompose the MR sinogram into multiscales in order to effectively denoise the image. Rabbani [11] proposed the spatially adaptive wavelet based method which uses the maximum a posteriori (MAP) criterion on a local Laplacian prior for denoising low SNR MR images in order to enhance the visual quality.

Other denoising MR images approaches have used the Non local means (NLM) filter introduced by Buades et al. [12] based on non-local averaging of all pixels in the image. Manjon et al. [13] modified the original NLM algorithm to denoise the multispectral MRI. In the multispectral sequences, the similarity measure can be obtained by combining information of various channels. Manjon et al. [14] proposed the unbiased NLM approach for MRI denoising. The unbiased NLM is obtained by subtracting the noise bias from the squared value of NLM. The main drawback of the NLM algorithm is computational burden due to its complexity especially on 3D MRI data. In order to overcome this, Coupe et al. [15] proposed an optimized blockwise NLM filter for denoising 3D MRI. This is achieved by the following steps: an automatic tuning of the smoothing parameter, a selection of the most relevant voxels for NL means computation, a blockwise, implementation and a parallelized computation. Manjon et al. [16] proposed the adaptive NLM filter for denoising MR images with spatially varying noise levels, such as those obtained by parallel imaging and surface coil acquisitions.

Recently, many of the popular de-noising algorithms suggested are based on wavelet thresholding [17], [18]. These approaches attempt to separate significant features/signals from noise in the frequency domain and simultaneously preserve them while removing noise. If the wavelet transform is applied on MR magnitude data directly, both the wavelet and the scaling coefficients of a noisy MRI image become biased estimates of their noise-free counterparts. Therefore, it was suggested [19] that the application of the wavelet transform on squared MR magnitude image data (which is noncentral chi-square distributed) would result in the wavelet coefficients no longer being biased estimates of their noise-free counterparts.

However, many of these algorithms remove the fine details and structure of the image in addition to the noise because of assumptions made about the frequency content of the image.

RBF neural networks have been widely used in image denoising, we can cite the works of Kaoru et al [20] who proposed a noise reduction filter that can reduce noise components without destroying important image information. The RBF network is then used to recover a high-quality image from the degraded version, and the regularization parameter is adjusted according to local image characteristics. A novel technique for blind image restoration and resolution enhancement based on radial basis function (RBF) neural network was used by Ping and Lei [21]. The RBF network gives a solution of the regularization problem often seen in function estimation with certain standard smoothness functional used as stabilizers. Li-yun et al. [22] have proposed a semi-blind defocused image deconvolution technique based on RBF neural network and iterative Wiener filtering.

In this paper, we propose a filtering method based RBF neural network to reduce a Gaussian noise from MRI image by minimizing a Total variation (TV) based error function. [23]. Indeed, the proposed filter uses the neighborhood information of a considered pixel as inputs and a single neuron producing the corresponding pixel in its output. The training of the proposed RBF network is achieved by the back-propagation algorithm for determining the optimal weights and centers.

The paper is organized with sections as follows. Section 2 presents the proposed filtering method, where the design of the used RBF neural network and its training algorithms are detailed. In Section 3, an experimental application of our RBFNN filter is illustrated to reduce noise from MRI images and its comparison with some known denoising methods. The last section is dedicated to some concluding remarks.

2. Proposed Filtering Method

We present a description of the RBF neural network Filter (RBFNNF) technique in this section. We start from the formulation of the image denoising problem by minimizing the TV model under constraints. Secondly, we propose the structure and training of RBFNNF used to minimizing an appropriate error function obtained from the total variation model in order to reduce the Gaussian noise.

2.1 Total Variation Model

Total Variation (TV) regularization has been extremely successful in a wide variety of denoising problems. It has been introduced for image denoising and reconstruction in a known paper of Rudin et al. [23] with the minimization of the following functional:

$$F(u)=\int_{\Omega}|D\quad|+\lambda\|u_0-u\|^2\ d\quad d\quad x\quad y \tag{1}$$

where $\int_{\Omega}|Du|$ represents the TV model of the image u. If the image u is regular, the equation (1) becomes only $\int_{\Omega}|\nabla u|\,dx$. Rudin et al. [1] considered that the noise which corrupted the image is distinguished from noiseless one by the size of total variation, which is defined as $\int_{\Omega}\sqrt{u_x^2+u_y^2}\;dx\,dy$, where Ω denotes the image domain u_x and u_y denote the corresponding partial differentiation.

To minimize (1) Rudin et al. [23] have applied the Euler-Lagrange equation where constraints are detailed in [24], they obtain the following equation:

$$\frac{\partial}{\partial x}\left(\frac{u_x}{\sqrt{u_x^2+u_y^2}}\right)+\frac{\partial}{\partial y}\left(\frac{u_y}{\sqrt{u_y^2+u_y^2}}\right)-\lambda(u-u_0)=0 \tag{2}$$

Where λ the Lagrange multiplier is given by:

$$\lambda=\frac{1}{2\sigma^2}\int\left[\sqrt{u_x^2+u_y^2}-\left(\frac{(u_0)_x u_x}{\sqrt{u_x^2+u_y^2}}\right)+\left(\frac{(u_0)_y u_y}{\sqrt{u_x^2+u_y^2}}\right)\right] \tag{3}$$

An image denoising problem can be transformed to an optimization problem. In our assumption, and from (3), we can formulate the image denoising problem as minimizing the following error function:

$$E(x,y)=\frac{\partial}{\partial x}\left(\frac{u_x}{\sqrt{u_x^2+u_y^2}}\right)+\frac{\partial}{\partial y}\left(\frac{u_y}{\sqrt{u_y^2+u_y^2}}\right)-\lambda\left(u-u_0\right) \tag{4}$$

where u_x and u_y are discretizations of the horizontal and vertical derivatives. A difficulty with TV is that, it has a derivative singularity when u is locally constant. To avoid this, some algorithms regularize TV by introducing an additional small parameter $\varepsilon>0$, $\sqrt{u_x^2+u_y^2+\varepsilon}$

2.2 RBF Neural Network Filter Design

To minimize (4), we propose to use a Radial Basic Function neural network (RBFNN). Generally, a RBFNN consists of three layers: the input layer, the RBF layer (hidden layer) and the output layer. The inputs of hidden layer are the linear combinations of scalar weights and the input vector $x[x_1,x_2,...,x_n]^T$, where the scalar weights are assigned unity values. Thus the whole input vector appears to each neuron in the hidden layer. The incoming vectors are mapping by the radial basis functions in each hidden node. The output layer yields a vector $y[y_1,y_2,...,y_m]^T$ for m outputs by linear combination of the outputs of the hidden nodes to produce the final output. Fig. 1 presents the structure of a single output RBFNN; the network output can be obtained by:

$$y=f(x)=\sum_{i=1}^{h}w_i\phi_i(x) \tag{5}$$

where f(x) is the final output, $\phi(.)$ denotes the radial basis function of the i-th hidden node, w_i denotes the hidden-to-output weight corresponding to the i-th hidden node, and h is the total number of hidden nodes.

A radial basis function is a multidimensional function that describes the distance between a given input vector and a pre-defined center vector. There are different types of radial basis function. A normalized Gaussian function usually is used as the radial basis function, it is given by:

$$\phi_i(x) = \exp\left[\frac{-\|x - c_i\|}{2\sigma_i^2}\right] \tag{6}$$

where c_i and σ_i denote the center and spread width of the i^{-th} node, respectively. The parameters of an RBFNN have to be initialized before the network is trained. The weights {wj, j =1 , 2 , . . . , M} can be initialized to either small random values or zeros. The initial centers can be determined, for example, by K-means clustering of a number of the input samples. Spread initial values σ_i could be chosen as the average of the nearest-neighbor distances among the initialized centers and trained later, or fixed during the training process; this parameter is generally fixed after multiple learning experimentations.

We take the assumption that the output of the RBFNNF is an image corresponding to the desired image u. So we provided as inputs to the RBFNNF the degraded image u_0. The output of the network is formulated by:

$$u = N(u_o, w_i, c_i) \tag{7}$$

where u_o is the noisy image represented by each intensity of pixel (x, y) and his neighbourhood (fig. 2).

The gray level of the output of each pixel u_0 (x,y) is calculated by:

$$N(x, y) = \left(\sum_{i=1}^{h} w_i \exp\left[\frac{-\|V_0(x, y) - c_i\|}{2\sigma_i^2}\right]\right) \tag{8}$$

where V_0 (x,y) is the gray level values of the neighbourhood pixels correspond to the pixel u_0 (x,y). The window size of the neighbourhood could be 3*3, 5*5, 7*7 ....

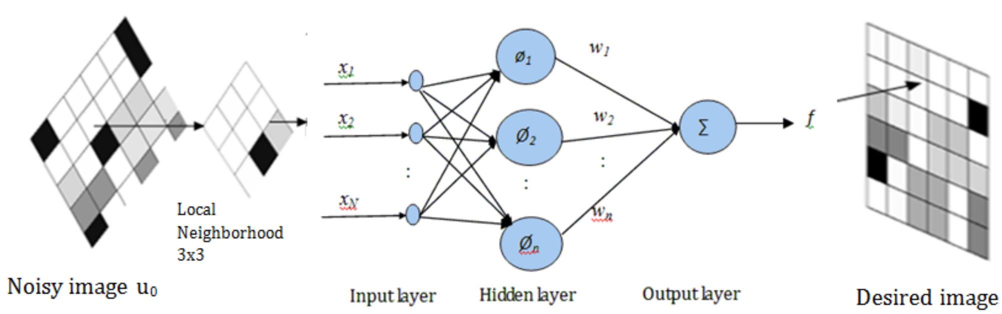


Fig. 1: RBFNN Filter of our approach

2.3 RBF Neural Network Filter Training

Once all pixels of the noisy image are presented to the network, an image u is obtained. We assume that this image satisfies the equation (2). This image is replaced in equation (3) to get λ, then in equation (4) for calculating the error function E which is provided by the network. The RBFNNF is trained by the back-probagation algorithm [25] until the maximum iterations number is reached or the obtained error is less than the convergence error threshold E_C. This algorithm adjusts the parameters w_i and c_i according to:

$$\begin{aligned} w_i(t+1) &= w_i(t) - \eta_1 \Delta w_i(t) \\ c_i(t+1) &= c_i(t) - \eta_2 \Delta c_i(t) \end{aligned} \tag{9}$$

Where $w_i(t+1)$ and $c_i(t+1)$ represent the values of parameters (weights and centres) of next iteration and $w_i(t)$ and $c_i(t)$ are their values for current iteration. η_1 and η_2 are positive learning rates.

The parameters variation Δw_i and Δc_i are obtained by minimization of error $E(x,y)$ presented in equation (4) using the following equation:

$$\Delta w_i = \sum_{x,y} \frac{\partial E(x,y)}{\partial w_i} \qquad (10)$$

$$\Delta c_i = \sum_{x,y} \frac{\partial E(x,y)}{\partial c_i}$$

Then

$$\frac{\partial E(x,y)}{\partial w_i} = \frac{\frac{\partial}{\partial x}\left(\frac{u_x}{\sqrt{u_x^2+u_y^2}}\right)}{\partial w_i} + \frac{\frac{\partial}{\partial y}\left(\frac{u_y}{\sqrt{u_x^2+u_y^2}}\right)}{\partial w_i} - \frac{\partial \lambda}{\partial w_i}(u-u_0) - \lambda \frac{\partial u}{\partial w_i} \qquad (11)$$

$$\frac{\partial E(x,y)}{\partial c_i} = \frac{\frac{\partial}{\partial x}\left(\frac{u_x}{\sqrt{u_x^2+u_y^2}}\right)}{\partial c_i} + \frac{\frac{\partial}{\partial y}\left(\frac{u_y}{\sqrt{u_x^2+u_y^2}}\right)}{\partial c_i} - \frac{\partial \lambda}{\partial c_i}(u-u_0) - \lambda \frac{\partial u}{\partial c_i} \qquad (12)$$

Training algorithm of the RBF network is carried out to find the optimal settings. Once converged, these settings are used to restore the image.

3. Experimental Results

In this section, we present some experimental results to evaluate the performance of RBFNNF approach. We also chose to compare the denoising performance of this approach with other denoising methods using their optimal parameters: minimizing TV model of Rudin et al. [23] and Wiener filter. To do this we used two MRI images.

To test objectively the performance of image denoising algorithm, the ISNR metric is often used. It represents the amount of noise removed from the degraded image. If ISNR increases, then the result of restoration is best. We also used the Normalized Mean Square Error (NMSE) as another measure of quality. If the value of NMSE decreases, the restoration is better.

In our experiments, The RBFNNF parameters are shown in table.1

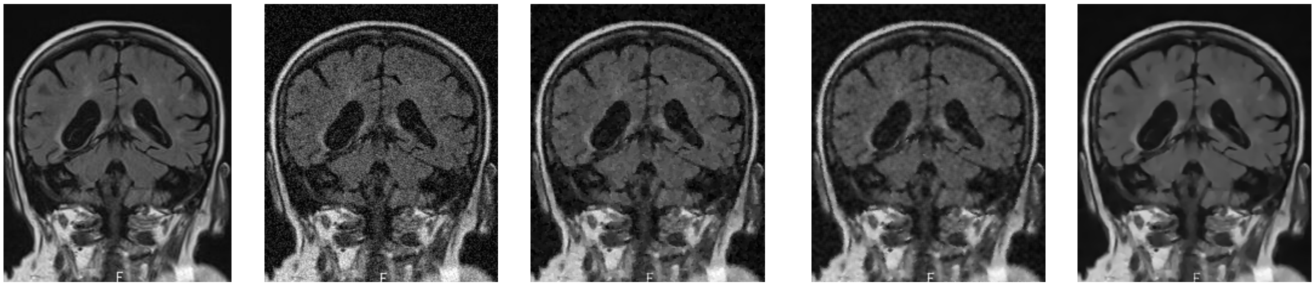
TABLE I: The MNNF used Parameters

Parameters	Value
Input layer size (window size)	3*3
Hidden neurons number	5
Output neurons	1
Convergence error threshold Ec	1e-06
Maximum iterations number	1000

The first denoising experiment is shown in Fig 2. For this experiment, using cerebral image of 193 × 236 pixels taken from public link: *http://www.thema-radiologie.fr/images/.orig/irm-cerebrale_IRM_cereb_coron_T2.jpg.*

We added a white Gaussian noise with zero mean and standard deviations $\sigma = 20$. The Fig 2.a and 2.b represent the original and noisy images respectively, Fig 2c, 2d 2e show the denoised image by TV approach, Wiener filter and RBFNNF approach respectively.

The proposed method (RBFNNF) gives a good visual quality with strong noise reduction and also more details are preserved.



a b c d e

Fig. 2: Brain image de-noising: (a) Original Image, (b) Gaussian degraded image with $\sigma = 20$, (c) de-noising image with TV approach (d) de-noising image with Wiener filter, (e) de-noising image with RBFNNF.

As second experimentation, we have applied our approach to denoising MRI of the knee in sagittal cut, of 512 x 512 pixels taken from http://www.ac-grenoble.fr/disciplines/sti-biotechnologies/articles.php?pg=131#IRM The results are given in Fig 3. For this experiment we increased the noise by adding white Gaussian noise with zero mean and standard deviations $\sigma = 25$.

The results presented show the good performance of our algorithm, especially the preservation of discontinuities. Moreover the geometric characteristics such as corners and edges and originals contrast are well restored.

a b c d e

Fig. 3: Knee image de-noising: (a) Original Image, (b) Gaussian degraded image with $\sigma = 25$, (c) de-noising image with TV approach (d) de-noising image with Wiener filter, (e) de-noising image with RBFNNF.

For purposes of comparison, the results of noise reduction by the three methods chosen are summarized in Table.II.

TABLE II: The ISNR and NMSE values of white Gaussian noise reduction.

Image	TV approach		Wiener filter		RBFNNF approach	
	ISNR	MNSE	ISNR	MNSE	ISNR	MNSE
brain	4.4023	0.0190	5.3941	0.0151	11,2422	0.0039
Knee	4.5375	0.0226	4.4934	0.0228	14.1881	0.0024

It is clear from result figures and Table.II that the proposed approach (RBFNNF) has enhanced the noise reduction ability of the neural filter when compared to the other method.

4. Conclusion

In this work, a novel technique is proposed for the de-noising of Magnetic Resonance Images using a neural network filter based on total variation regularization. A RBF neural network is used to reduce the Gaussian noise from the degraded image by minimizing a TV based error function. The training of the RBFNN filter is carried out by back-propagation algorithm to adjust the nodes centers and the hidden layer weights.

The performance of the proposed denoising filter is compared with TV restoration method and wiener filter. This filtering method tends to produce good denoised image not only in terms of visual perception but also in terms of the quality metrics. The edge preserving property is clearly an advantage of the proposed method.

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