

Comparison of the Design of Reinforced Concrete Elements under the Shear Force According to Albanian Normative

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Abstract: In this work, the authors, presents the findings obtained from the analysis of calculating the shear strength of reinforced concrete elements under the shear force. The chose element, a reinforced concrete beam, is designed according to Albanian Normative with two methods, allowable stress design method and limit states design method. Furthermore, is made a comparison between the analysis results. In the beginning a presentation is made with the theoretical solution of the problem and after that the comparison is based on numerical solution. The conclusions are followed from the recommendations given in the end of this work.

Keywords: allowable stress design method, limit state design method, design of elements under shear force, Albanian Normative

1. Introduction

The design of reinforced concrete structures, in different time periods, is performed in accordance with approved technical normative, using three methods [1]:

- allowable stress design method or classic method
- rupture method
- limit state design method

In years '60 – '70 of the last century, in Albania, the allowable stress design method was used for the design of motorway or railway bridges and also for the design of hydro technical structures where the cracking where not allowed. The rupture method was used for the design of industrial and civil buildings. The limit state design method was used without restrictions for reinforced concrete structures and also for the prestressed concrete structures. This method is the one that is used actually in our country. The comparison of the results of the calculation of the bearing shear force of the same element according to the two methods shows the usage advantage of the limit state design method against allowable stress design method.

2. Design Methods of Reinforced Concrete Elements According to Albanian Normative

2.1. Symbols According the Two Methods

The symbols between {...} belong to the allowable stress design method. Only the symbols that differ are shown. The same symbols are not shown.

a_{sw} – reinforcing steel area of one of the legs of the stirrup ; $\{f_{a,st}\}$

n – number of stirrup legs, $n = 2,3,4\dots$

$A_{sw} = n \cdot a_{sw}$ – reinforcing steel area for one stirrup; $\{F_{a,st} = n \cdot f_{a,st}\}$

A_{sinc} – inclined reinforcing steel area; $\{F_{a,0}\}$

s_w – distance between stirrups $\{a_x\}$

Q_b – concrete shear strength; $\{[Q_b]\}$

Q_{sw} – stirrups shear strength $\{[Q_{st}]\}$

$Q_{As,inc}$ – inclined reinforcement shear strength; $\{[Q_0]\}$

2.2. Allowable Stress Design Method

In Albania this is the first and the oldest method and for this reason is also known as the classic method. In a flexural reinforced concrete element, together with the normal stresses there are also the tangential stresses. From the Constructions Science is known that for a homogeny element, tangential stresses are found with the following formula:

$$\tau = (Q \cdot S) / (I \cdot b) \quad (2.2.1)$$

Q – shear force

S – static moment of the surface over the fibres where the tangential stresses are calculated, against the neutral axis.

I – moment of inertia against the neutral axis

b – cross section width

For concrete elements in order to determine the maximum tangential stresses is used (2.2.2)

$$\tau = Q / (b \cdot z) \quad (2.2.2)$$

$$z \approx 0.9 \cdot h_0 \quad (2.2.3)$$

h_0 – cross section effective depth

Main stresses:

$$\sigma_{kr} = \frac{\sigma}{2} \pm \sqrt{\frac{\sigma^2}{4} + \tau^2} \quad (2.2.4)$$

The angel formed with horizontal axis:

$$\operatorname{tg}2\alpha = -\frac{2 \cdot \tau}{\sigma} \quad (2.2.5)$$

More important are the main tensile stresses because of the low tensile strength of concrete. On neutral axis $\sigma = 0$, $\alpha = 45^\circ$, $\sigma_{kr} = \pm\tau$. Condition (2.2.6) must be fulfilled. If is not fulfilled then the cross section dimensions or the concrete strength must be increased.

$$\sigma_{kr} \leq \sigma_{bt} \quad (2.2.6)$$

If condition (2.2.7) is fulfilled, then there is no need to check the element for shear.

$$\sigma_{kr} \leq \sigma_{bt,1} \quad (2.2.7)$$

Different from many European Normative, the Albanian Normative, together with the stirrups and inclined reinforcement strength take in consideration also the role of the concrete in the shear strength of the element. Is taken in consideration an element with a rectangular cross section with dimension $b \times h$. Main stresses bear from the concrete:

$$\sigma_{kr} = \sigma_{bt,2} \quad (2.2.8)$$

Values of σ_{bt} , $\sigma_{bt,1}$, $\sigma_{bt,2}$ are given from the Normative in accordance with the concrete grade. Shear strength of stirrups, expressed in length unit:

$$[q_{st}] = (n \cdot f_{a,st} \cdot [\sigma_a]) / a_x \quad (2.2.9)$$

n – number of stirrup legs

$[\sigma_a]$ – allowable stresses in the reinforcing steel

Main stresses bear from stirrups:

$$\sigma_{kr} = \frac{n \cdot f_{a,st} \cdot [\sigma_a]}{b \cdot a_x} \quad (2.2.10)$$

Shear strength of inclined reinforcement:

$$[Q_0] = F_{a,0} \cdot [\sigma_a] \cdot \sin \alpha \quad (2.2.11)$$

α – inclination angle of the inclined reinforcement, usually 45°

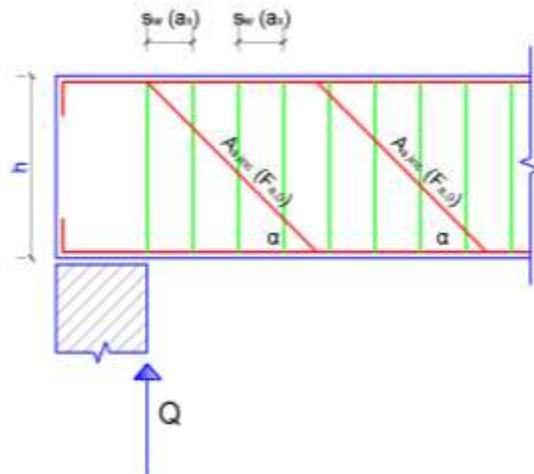


Fig. 2.2.1: Stirrups and inclined reinforcing in a beam

2.3. Limit State Design Method

According to Albanian normative [7], [8], [9]:

$$\tau = Q / (b \cdot h_0) \quad (2.3.1)$$

Minimum shear strength of concrete:

$$Q_{b,min} = \varphi_b \cdot b \cdot h_0 \cdot R_{bt} \quad (2.3.2)$$

φ_b – take in consideration the factors that affects the concrete work, usually $\varphi_b=0.6$

R_{bt} – tensile strength of concrete

If condition (2.3.3) is fulfilled, then there is no need to check the element for shear.

$$Q \leq Q_{b,min} \quad (2.3.3)$$

Condition (2.3.4) must be fulfilled, if it is not, then the cross section dimensions or the concrete class must be increased.

$$Q \leq 0.25 \cdot b \cdot h_0 \cdot R_b \quad (2.3.4)$$

R_b – concrete compression strength

If condition (2.3.5) is fulfilled, then the element must be designed from shear force:

$$Q_{b,min} < Q \leq 0.25 \cdot b \cdot h_0 \cdot R_b \quad (2.3.5)$$

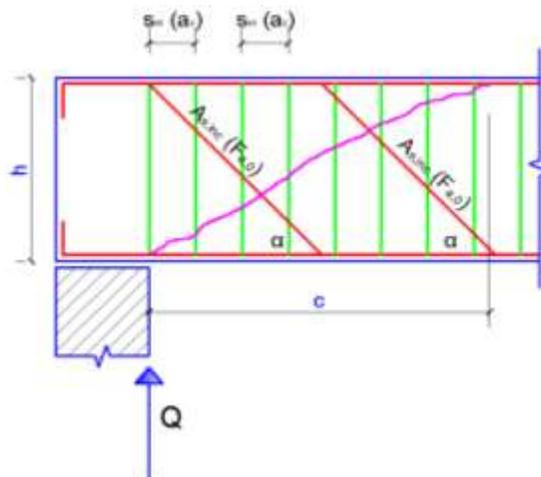


Fig. 2.3.1: Shear crack on a beam support.

For a random inclined crack, with a horizontal length “c” the following condition must be fulfilled:

$$Q \leq Q_b + Q_{sw} + Q_{As,inc} \quad (2.3.6)$$

$$Q_b = (\varphi_{b2} \cdot b \cdot h_0^2 \cdot R_{bt}) / c \quad (2.3.7)$$

φ_{b2} – coefficient, usually $\varphi_{b2}=2$

$$Q_{sw} = (n \cdot a_{sw} \cdot R_{sw} \cdot c) / s_w \quad (2.3.8)$$

R_{sw} – reinforcing steel shear strength, $R_{sw}=0.8 \cdot R_s$

$$Q_{As,inc} = \Sigma A_{s,inc} \cdot R_{sw} \cdot \sin\alpha \quad (2.3.9)$$

The most dangerous cut, with a minimum shear strength have a horizontal length “ c_0 ”.

$$c_0 = \sqrt{\frac{(\varphi_{b2} \cdot b \cdot h_0^2 \cdot R_{bt}) \cdot s_w}{n \cdot a_{sw} \cdot R_{sw}}} \quad (2.3.10)$$

If we substitute (2.3.10) in equations (2.3.7) and (2.3.8), we will have:

$$(Q_b + Q_{sw})_{min} = \sqrt{\frac{(4 \cdot \varphi_{b2} \cdot b \cdot h_0^2 \cdot R_{bt}) \cdot (n \cdot a_{sw} \cdot R_{sw})}{s_w}} \quad (2.3.11)$$

2.4. Results of the Calculations of Flexural Reinforced Concrete Elements

Numerical example 1

A beam under flexural action is analyzed. Rectangular cross section with $b = 30\text{cm}$ and $h = 60\text{cm}$, $a = a' = 4\text{cm}$, $h_0 = h - a = 60 - 4 = 56\text{cm}$. According to allowable stress design method concrete is of a grade M 300 (cubic resistance), $\sigma_{bt} = 30\text{daN/cm}^2$, $\sigma_{bt1} = 12\text{daN/cm}^2$, $\sigma_{bt2} = 7.5\text{daN/cm}^2$; steel Ç.5, $[\sigma_a] = 1600\text{daN/cm}^2$; According to limit state design method the concrete is of class B30 (cubic resistance), $R_b = 160\text{daN/cm}^2$; steel Ç.5, $R_s = 2400\text{daN/cm}^2$, $R_{sw} = 1920\text{daN/cm}^2$; stirrups $\Phi 8$, $s_w(a_x) = 15\text{ cm}$, $n = 2$. Inclined reinforcement as in figure

Determine, with the two methods, the shear strength of the beam.

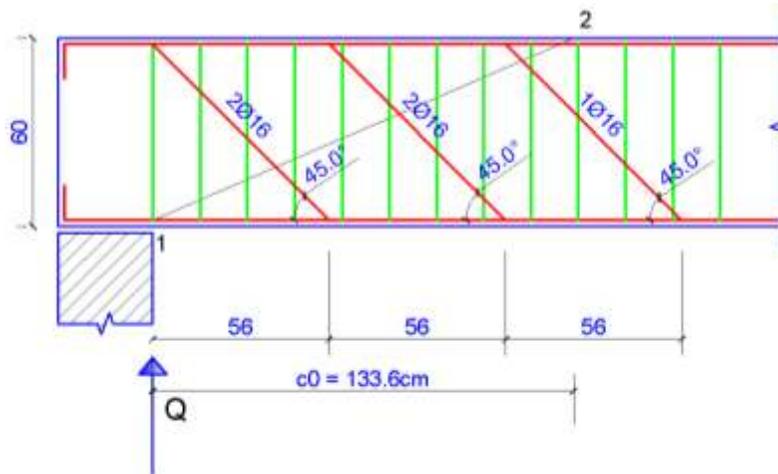


Fig. 2.4.1: Beam support reinforcement

a) Limit State Design Method

With help of equation (2.3.10) determine $c_0 = 133.6\text{cm}$.

With help of equation (2.3.7) determine $Q_b = 17143\text{daN}$

With help of equation (2.3.8) determine $Q_{sw} = 17139\text{daN}$

With help of equation (2.3.11) determine $(Q_b + Q_{sw})_{min} = 34282\text{daN}$

Crack 1 – 2 is the most dangerous. Its horizontal length is c_0 . In this crack there are in total $5\Phi 16$, so $\Sigma A_{s,inc} = 10.05\text{cm}^2$. With help of equation (2.3.9) determine $Q_{As,inc} = 13644\text{daN}$. Total shear force $Q_b + Q_{sw} + Q_{As,inc} = 17143 + 17139 + 13644 = 47926\text{daN}$.

b) Allowable Stress Design Method

For crack 1 – 2 we have:

With help of equation (2.2.2), (2.2.3), (2.2.8) determine the shear strength of concrete $[Q_b] = \sigma_{bt,2} \cdot b \cdot 0.9 \cdot h_0 = 11340\text{daN}$.

Beginning from (2.2.9) we find the shear strength of stirrups $[Q_{st}] = [q_{st}] \cdot c_0 = 14282\text{daN}$ (2.4.1)

With help of equation (2.2.11) determine $[Q_0]=11368\text{daN}$

Total shear force $[Q_b] + [Q_{st}] + [Q_0] = 11340 + 14282 + 11368 = 36990\text{daN}$.

The concrete shear strength, calculated with limit state method, results higher that the respective shear strength calculated with allowable stress design method (A.S.D) (+51%).

Stirrups shear strength calculated with limit state method; results higher that the respective shear strength calculated with allowable stress design method (A.S.D) (+20%).

Inclined reinforcement shear strength calculated with limit state method, results higher that the respective shear strength calculated with allowable stress design method (A.S.D) (+20%).

Total shear strength calculated with limit state method, results higher that the respective shear strength calculated with allowable stress design method (A.S.D) (+29.5%).

2.5. Calculations Analysis Results

To make possible the comparison of the calculation results, the elements are considered in the same conditions. The same class of concrete and steel is accepted, the same quantity of inclined reinforcement, the same distance between stirrups, the same stirrups diameter, the same cross section dimensions. Graphically is showed the connection between the shear strength of concrete, stirrups, inclined reinforcement, total shear force and factors as: cross section width, cross section height, allowable stresses of concrete, of reinforcing steel, of inclined reinforcement, distance between stirrups, and quantity of stirrups reinforcement.

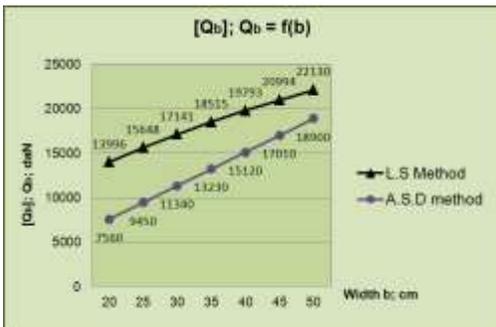


Fig. 2.5.1: Concrete shear strength and width relationship

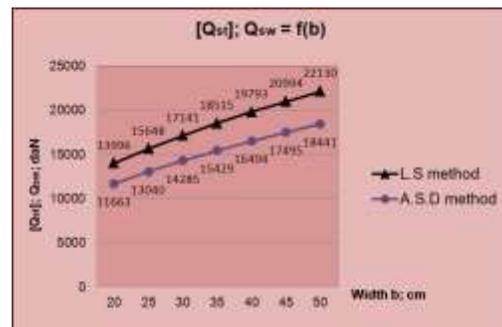


Fig. 2.5.2: Stirrups shear strength and width relationship

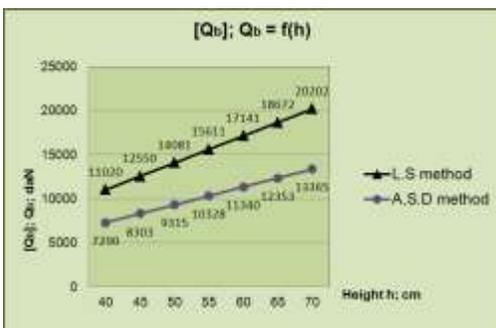


Fig. 2.5.3: Concrete shear strength and height relationship

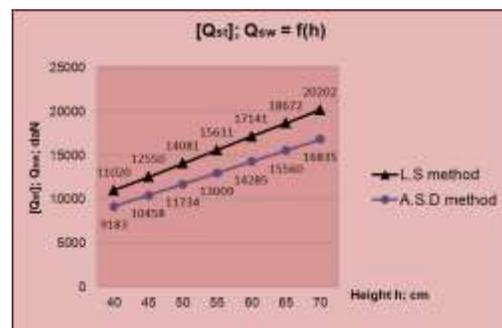


Fig. 2.5.4: Stirrups shear strength and height relationship

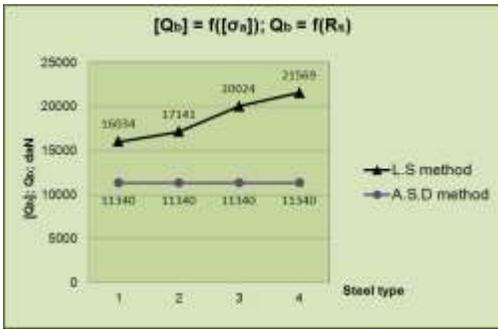


Fig. 2.5.5: Concrete shear strength and steel relationship

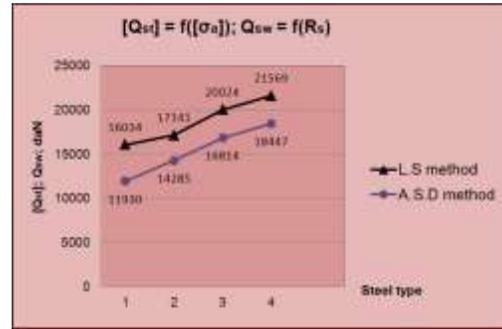


Fig. 2.5.6: Stirrup shear strength and steel relationship

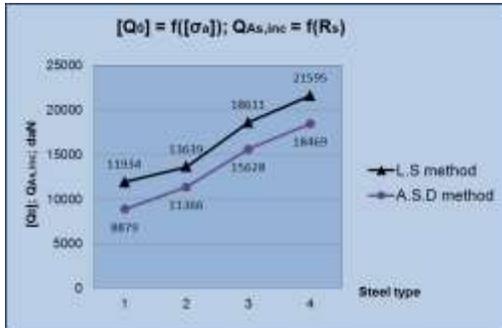


Fig. 2.5.7: Inclined reinforcement shear strength and steel relationship

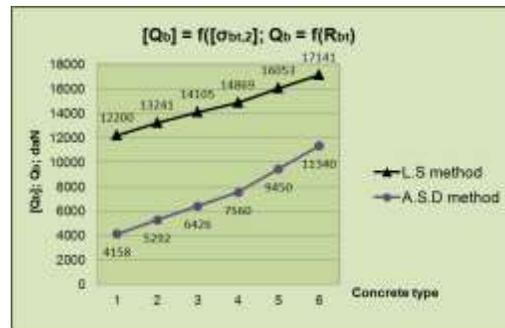


Fig. 2.5.8: Concrete shear strength and concrete type relationship

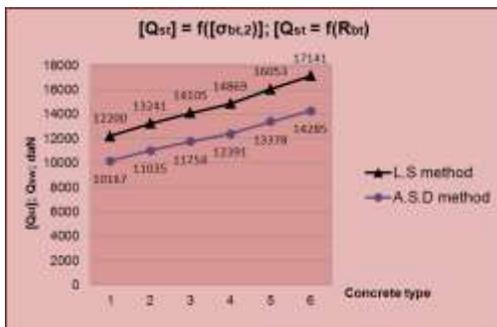


Fig. 2.5.9: Stirrup shear strength and concrete type relationship

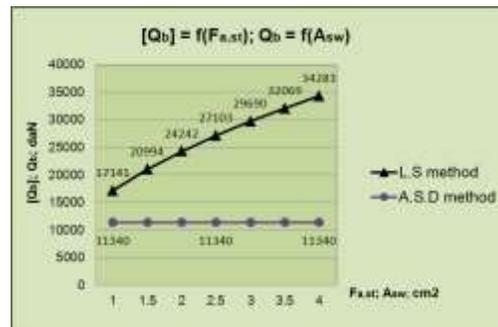


Fig. 2.5.10: Concrete shear strength and stirrup reinforcement area relationship

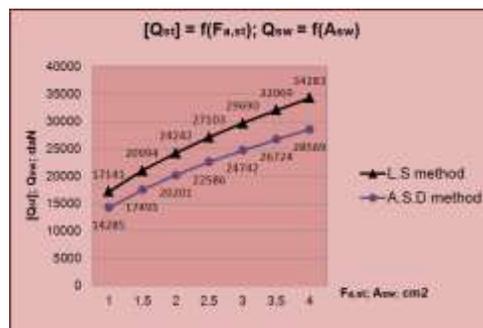


Fig. 2.5.11: Stirrup shear strength and stirrup reinforcement area relationship

3. Conclusions

- In all the cases shear strength of an element, calculated with the limit state method (L.S) is higher than the one calculated with allowable stress design method (A.S.D). This is an expected result knowing that the L.S method uses in a better way the concrete and reinforcing steel work after elastic phase. See all the figures.
- Increasing of the cross section width 'b' brings in the increasing of the shear strength of concrete, calculated with both the methods. See figure 2.5.1
- When the width is increased 2.5 times, $[Q_b]$ is increased 1.58 times.
- Width increasing affects more the $[Q_b]$. The diagram is more inclined. See figure 2.5.1
- Width increasing does not affect directly $[Q_{st}]$ and Q_{sw} , but width increasing brings the increasing of "c₀" and increase $[Q_{st}]$ and Q_{sw} . See figure 2.5.2 and equations (2.3.8), (2.3.10) and (2.4.1)
- A 2.5 times increasing of width brings an increasing of 1.58 times "c₀" and also 1.58 times $[Q_{st}]$ and Q_{sw} .
- Width increasing has almost the same effect to both $[Q_{st}]$ and Q_{sw} . The diagrams are almost parallel. See figure 2.5.2.
- Width increasing does not affect directly $[Q_0]$ and $Q_{As,inc.}$, but "c₀" increasing maybe can include more reinforcing inside the cut that is taken in consideration, increasing indirectly $[Q_0]$ and $Q_{As,inc}$
- Increasing of cross section height "h" brings the increasing of concrete shear strength, calculated with both the methods. See figure 2.5.3.
- A 1.75 times increasing of height brings a 1.83 times increasing of $[Q_b]$ and Q_b .
- Height increasing affects more Q_b . The diagram is more inclined. See figure 2.5.3
- Height increasing does not affect directly $[Q_{st}]$ and Q_{sw} , but height increasing brings the increasing of "c₀" and increase $[Q_{st}]$ and Q_{sw} . See figure 2.5.4 and equations (2.3.8), (2.3.10) and (2.4.1)
- A 1.75 times increasing of height brings an increasing of 1.83 times "c₀" and also 1.83 times $[Q_{st}]$ and Q_{sw} .
- Height increasing has almost the same effect to both $[Q_{st}]$ and Q_{sw} . The diagrams are almost parallel. See figure 2.5.4.
- Height increasing does not affect directly $[Q_0]$ and $Q_{As,inc.}$, but "c₀" increasing maybe can include more reinforcing inside the cut that is taken in consideration, increasing indirectly $[Q_0]$ and $Q_{As,inc}$
- Increasing of steel tensile strength or increasing of $[\sigma_a]$, does not affect the $[Q_b]$
- Increasing of R_s brings the increasing of Q_b because "c₀" is decreased. See figure 2.5.5 and equations (2.3.10), (2.3.7). Changing the type of steel from 1 to 4, R_s is increased.
- When R_s is increased 1.8 times, "c₀" is decreased 1.34 times and Q_b is increased 1.34 times
- Increasing of $[\sigma_a]$ or R_s brings the increasing of $[Q_{st}]$ and Q_{sw} . See figure 2.5.6. The effect is almost the same to $[Q_{st}]$ and Q_{sw} . The diagrams are almost parallel.
- Increasing of $[\sigma_a]$ or R_s brings the increasing of $[Q_0]$ and $Q_{As,inc.}$ See figure 2.5.7. The effect is almost the same to $[Q_{st}]$ and Q_{sw} . The diagrams are almost parallel.
- Increasing of concrete grade brings the increasing of $[Q_b]$ and Q_b . See figure 2.5.8.
- The differences between the values of the forces calculated according to both methods are evident. This because of usage after the elastic phase of the material by the L.S method. The effect is almost the same. The diagrams are almost parallel.
- Concrete grade increasing does not affect directly $[Q_{st}]$ and Q_{sw} , but concrete grade increasing brings the increasing of "c₀" and increase $[Q_{st}]$ and Q_{sw} . See figure 2.5.9 and equations (2.3.8), (2.3.10) and (2.4.1).
- Concrete grade increasing does not affect directly $[Q_0]$ and $Q_{As,inc.}$, but "c₀" increasing maybe can include more inclined reinforcing inside the cut that is taken in consideration, increasing indirectly $[Q_0]$ and $Q_{As,inc}$.
- Increasing of stirrup reinforcing, or $F_{a,st}$, does not affect $[Q_b]$.

- Increasing of A_{sw} brings the increasing of Q_b because the decreasing of “ c_0 ”. See figure 2.5.10 and equation (2.3.10).
- A 4 time increasing of A_{sw} brings a 2 time decreasing of “ c_0 ” and a 2 time increasing of Q_b .
- Increasing of $F_{a,st}$ or A_{sw} affect in the same way $[Q_{st}]$ and Q_{sw} . See figure 2.5.11.
- When $F_{a,st}$, or A_{sw} are increased 4 time then $[Q_{st}]$ and Q_{sw} are increased 2 time.
- Increasing of $F_{a,st}$, or A_{sw} affect in the same way $[Q_{st}]$ and Q_{sw} . The diagrams are almost parallel.
- $F_{a,st}$, or A_{sw} increasing does not affect directly $[Q_0]$ and $Q_{As,inc.}$, but $F_{a,st}$, or A_{sw} increasing brings the decreasing of ‘ c_0 ’ and maybe can include more inclined reinforcing inside the cut that is taken in consideration, This would cause the decreasing of $[Q_0]$ and $Q_{As,inc.}$
- As a conclusion, we can say, that the transition from the allowable stress design method to the limit state method, for the shear force design, according to Albanian Normative, brings the decreasing of the reinforcing and concrete quantity, decreasing the cost.

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