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Reliability Analysis of a Fuel Supply System in Automobile Engine

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Abstract: *The present paper deals with the analysis of a fuel supply system in an automobile engine of a four wheeler which is having both the option of fuel i.e. PETROL and CNG. Since CNG is cheaper than petrol so the priority is given to consume CNG as compared to petrol. An automatic switch is used to start petrol supply at the time of failure of CNG supply. Using regenerative point technique with Markov renewal process, the reliability characteristics which are useful to system designers are obtained.*

Keywords: *Reliability, Redundancy, Repair Time, Transition Probability, Regenerative Points, Markov Renewal Process.*

1. Introduction

Many authors working in the field of system reliability have analysed various engineering systems by using different sets of assumptions. Several authors developed and analysed many hypothetical systems, but there is a need to work on the systems which are used very commonly and related to the real practical situation. Goel et al. (1985) provided studied a system with intermittent repair and inspection under abnormal weather and also provided cost benefit analysis of the system. Joorel (1990) calculated MTSF and availability analysis of a complex system composed of two sub-systems in series subject to random shocks with single repair facility. Reddy and Rao (1993) showed how to optimize the parallel systems subject to two modes of failure and repair provision.

Agnihotri et al. (2008) studied and carried out reliability analysis of a system of boiler used in Readymade garment industry. Chander and Singh (2009) studied a reliability model of 2-out-of-3 redundant system subject to degradation after repair.

Agnihotri et al. (2010) presented reliability analysis of a system of wheels in a four wheeler. Marko Gerbec (2010) introduced reliability analysis of a natural gas pressure regularity installation. EL-Damcese and Tamraz, N.S. (2012) presented analysis of a parallel redundant system with different failure mode.

Keeping the above in view, we in the present chapter analysed a system of fuel supply system in an automobile engine of a four wheeler which is having both the option of fuel i.e. PETROL and CNG. Since CNG is cheaper than petrol so the priority is given to consume CNG as compared to petrol. An automatic switch is used to start petrol supply at the time of failure of CNG supply.

Using regenerative point technique with Markov renewal process, the following reliability characteristics which are useful to system designers are obtained.

(1) Transition and steady state transition probabilities

(2) Mean Sojourn times in various states

(3) Mean time to system failure (MTSF)

(4) Point wise and Steady state availability of the system

(5) Expected Busy period of the repairman in (0,t]

(6) Expected number of visits by the repairman in (0,t]

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2. Model Description And Assumptions

- The fuel system of an automobile engine consists of both the facilities i.e. CNG and petrol supply. Since CNG is cheaper than petrol so the priority will be given to CNG for both “use”, and “repair”, to run the four wheeler.
- Initially the four wheeler starts operation with CNG and the petrol facility is kept as standby. Both CNG and petrol has only two modes i.e. normal and failure.
- Whenever CNG system fails petrol starts operation instantaneously with the help of an automatic switch, provided that the switch is good at the time of need, otherwise the petrol supply will wait until the switch is repaired. Probability that the automatic switch will be in good position at the time of need is fixed.
- Since the consumption of petrol is too costly as compared to CNG, so there is a provision of rest to petrol supply after continuous working for a random amount of time.
- A single repair facility is available in the system to repair the jobs related to CNG, petrol supply and automatic switch.
- The failure time distributions of CNG and petrol supply are exponential with different parameters while the repair time distributions of jobs related to CNG, petrol and automatic switch are arbitrary. Also the distribution of time after which rest will be provided to petrol and its rest time are exponential with different parameter.

3. Notation and Symbols

C_{NS} : CNG is in normal supply

P_{NS} : Petrol is in normal supply

P_S : Petrol supply is in standby position

C_{fr} : Failed CNG supply under repair

C_{fwr} : Failed CNG supply waiting for repair

P_{rest} : Petrol is in rest position

P_{fr} : Failed petrol supply under repair

P_{fwr} : Failed petrol supply waiting for repair

AS_r : Automatic switch under repair

□ : Constant failure rate of CNG supply

□ : Constant failure rate of petrol supply

□ : Constant rate of completing rest for petrol supply

□ : Constant rate of time after which rest is to be provide to petrol supply

g(.), G(.) : pdf and cdf of repair time distribution of CNG supply

h(.), H(.) : pdf and cdf of repair time distribution of petrol supply

k(.), K(.) : pdf and cdf of repair time distribution of automatic switch

p(=1-q) : Probability that automatic switch will be in good position at the time of need.

Using the above notation and symbols the following are the possible states of the fuel system of an automobile engine system:

<u>Up States</u>

S_0 □ (C_{NS}, P_S) S_1 □ (C_{fr}, P_{NS}) S_5 □ (C_{NS}, P_{rest})

S_6 □ (C_{NS}, P_{fr})

<u>Down States</u>

S_2 □ (C_{fwr}, P_S, AS_r) S_3 □ (C_{fr}, P_{rest}) S_4 □ (C_{fr}, P_{fwr})

The transitions between the above states are shown in Figure.1.

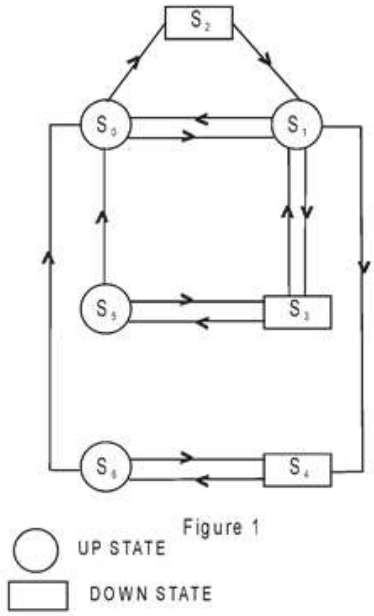

Figure 1

4. Transition Probabilities

Let T_0 (=0), T_1,T_2,.... be the epochs at which the system enters the states $S_i \in E$. Let X_n denotes the state entered at epoch T_{n+1} i.e. just after the transition of T_n. Then $\{T_n,X_n\}$ constitutes a Markov-renewal process with state space E and

$$Q_{ik}(t) = \Pr[X_{n+1} = S_k, T_{n+1} - T_n \leq t \mid X_n = S_i]$$

is semi Markov-Kernal over E. The stochastic matrix of the embedded Markov chain is

$$P = p_{ik} = \lim_{t\to\infty} Q_{ik}(t) = Q(\infty)$$

Thus

$p_{01} = p$ $\qquad$ $p_{02} = q$

$p_{10} = \tilde{G}(\beta+\delta)$ $\qquad$ $p_{13} = \dfrac{\delta}{\beta+\delta}\tilde{G}(\beta+\delta)$

$p_{14} = p^{(4)}_{16} = \dfrac{\beta}{\beta+\delta}[1-\tilde{G}(\beta+\delta)]$ $\qquad$ $p_{21} = p_{46} = 1$

$$p^{(1)}_{30} = \frac{\gamma}{\gamma-\beta-\delta}[\tilde{G}(\beta+\delta)-\tilde{G}(\gamma)]$$

$$p^{(1)}_{33} = \frac{\delta}{(\gamma-\beta-\delta)(\beta+\delta)}[(\gamma-\beta-\delta) - \gamma\tilde{G}(\beta+\delta) + (\beta+\delta)\tilde{G}(\gamma)]$$

$$p^{(1,4)}_{36} = \frac{\beta}{(\gamma-\beta-\delta)(\beta+\delta)}[(\gamma-\beta-\delta) - \gamma\tilde{G}(\beta+\delta) + (\beta+\delta)\tilde{G}(\gamma)]$$

$$p_{35} = \tilde{G}(\gamma) \qquad p_{50} = \frac{\gamma}{\alpha+\gamma}$$

$$p_{53} = \frac{\alpha}{\alpha+\gamma} \qquad p_{60} = 1 - \tilde{H}(\alpha)$$

$$p_{64} = \tilde{H}(\alpha) \qquad \text{….(3-16)}$$

5. Mean Sojourn Times

The mean time taken by the system in a particular state S_i before transiting to any other state is known as mean sojourn time and is defined as

$$\mu_i = {}_0\int^{\infty} P[T>t]\, dt$$

where T is the time of stay in state S_i by the system.

To calculate mean sojourn time μ_I in state S_i, we assume that so long as the system is in state S_i, it will not transit to any other state. Therefore;

$$\mu_0 = \frac{1}{\alpha} \qquad \mu_1 = \frac{1}{\beta+\delta}[1 - \tilde{G}(\beta+\delta)]$$

$$\mu_2 = {}_0\int^{\infty} \bar{K}(t)dt = m_1 \text{ (say)} \qquad \mu_3 = \frac{1}{\gamma}[1 - \tilde{G}(\gamma)]$$

$$\mu_4 = {}_0\int^{\infty} \bar{G}(t)dt = m_2 \text{ (say)} \qquad \mu_5 = \frac{1}{\alpha+\gamma}$$

$$\mu_6 = \frac{1}{\alpha}[1 - \tilde{H}(\alpha)] \qquad \text{….(17-23)}$$

<u>Contribution to Mean Sojourn Time</u>

For the contribution to mean sojourn time in state $S_i \in E$ and non-regenerative state occurs, before transiting to $S_j \in E$, i.e.,

$$m_{ij} = -\int t.q_{ij}(t)\, dt = -q'^{*}_{ij}(0)$$

Therefore,

$$m_{01} = p.\,{}_0\int^{\infty} t.\alpha e^{-\alpha t}\, dt \qquad m_{02} = q.\,{}_0\int^{\infty} t.\alpha e^{-\alpha t}\, dt$$

$$m_{10} = {}_0\int^{\infty} t.e^{-(\beta+\delta)t}\, g(t)dt \qquad m_{13} = \delta.{}_0\int^{\infty} t.e^{-(\beta+\delta)t}\, \bar{G}(t)dt$$

$$m_{14} = {}_0\int^{\infty} t.\beta e^{-(\beta+\delta)t}\, \bar{G}(t)dt \qquad m_{21} = {}_0\int^{\infty} t.k(t)dt$$

$$m_{35} = {}_0\int^{\infty} t.e^{-\gamma t}\, g(t)dt \qquad m_{46} = {}_0\int^{\infty} t.g(t)dt$$

$$m_{50} = {}_0\int^{\infty} t.\gamma e^{-(\alpha+\gamma)t}\, dt \qquad m_{53} = {}_0\int^{\infty} t.\alpha e^{-(\alpha+\gamma)t}\, dt$$

$$m_{60} = {}_0\int^{\infty} t.e^{-\alpha t}\, h(t)dt \qquad m_{64} = {}_0\int^{\infty} t.\alpha e^{-\alpha t}\, \bar{H}(t)dt$$

$$m^{(4)}_{16} = \frac{\beta}{\beta+\delta}\ {}_0\int^\infty t.(1 - e^{-(\beta+\delta)t})\ dG(t)$$

$$m^{(1)}_{30} = \frac{\gamma}{\gamma-\beta-\delta}\ {}_0\int^\infty t.(e^{-(\beta+\delta)t} - e^{-\gamma t})\ dG(t)$$

$$m^{(1)}_{33} = \frac{\delta.\gamma}{\gamma-\beta-\delta}\ {}_0\int^\infty t.(e^{-(\beta+\delta)t} - e^{-\gamma t})\ \overline{G}(t)\ dt$$

$$m^{(1,4)}_{36} = \frac{\beta}{(\beta+\delta)(\gamma-\beta-\delta)}\ {}_0\int^\infty t.[(\gamma\text{-}\beta\text{-}\delta) - e^{-(\beta+\delta)t} - (\beta+\delta)e^{-\gamma t}\ ]dG(t)$$

….(24-39)

Hence using (24-39) the following relations can be verified as follows

$$m_{01} + m_{02} = p.\ {}_0\int^\infty t.\alpha e^{-\alpha t}\ dt\ + q.\ {}_0\int^\infty t.\alpha e^{-\alpha t}\ dt$$

$$= {}_0\int^\infty t.\alpha e^{-\alpha t}\ dt = \frac{1}{\alpha} = \mu_0$$

$$m_{10} + m_{13} + m^{(4)}_{16} = {}_0\int^\infty t.e^{-(\beta+\delta)t}\ dG(t) + \delta.{}_0\int^\infty t.e^{-(\beta+\delta)t}\ \overline{G}(t)dt$$

$$+ \frac{\beta}{\beta+\delta}\ {}_0\int^\infty t.(1 - e^{-(\beta+\delta)t})\ dG(t) = n_1\ \text{(say)}$$

$$m_{21} = {}_0\int^\infty t.k(t)dt = \mu_2 = m_1$$

$$m_{35} + m^{(1)}_{30} + m^{(1)}_{33} + m^{(1,4)}_{36} = {}_0\int^\infty t.e^{-\gamma t}\ dG(t)$$

$$+ \frac{\gamma}{\gamma-\beta-\delta}\ {}_0\int^\infty t.(e^{-(\beta+\delta)t} - e^{-\gamma t})\ dG(t)$$

$$+ \frac{\delta.\gamma}{\gamma-\beta-\delta}\ {}_0\int^\infty t.(e^{-(\beta+\delta)t} - e^{-\gamma t})\ \overline{G}(t)\ dt$$

$$+ \frac{\beta}{(\beta+\delta)(\gamma-\beta-\delta)}\ {}_0\int^\infty t.[(\gamma\text{-}\beta\text{-}\delta) - e^{-(\beta+\delta)t} - (\beta+\delta)e^{-\gamma t}\ ]dG(t) = n_2\ \text{(say)}$$

$$m_{46} = {}_0\int^\infty t.g(t)dt = \mu_4 = m_2$$

$$m_{50} + m_{53} = {}_0\int^\infty t.\gamma e^{-(\alpha+\gamma)t}\ dt + {}_0\int^\infty t.\alpha e^{-(\alpha+\gamma)t}\ dt = \frac{1}{\alpha+\gamma} = \mu_5$$

$$m_{60} + m_{64} = {}_0\int^\infty t.e^{-\alpha t}\ dH(t) + {}_0\int^\infty t.\alpha e^{-\alpha t}\ \overline{H}(t)dt$$

$$= \frac{1}{\alpha}[1 - \widetilde{H}\ (\alpha)] = \mu_6$$

….(40-46)

6. Reliability and Mean Time to System Failure

To obtain the reliability of the system we regard all the failed states as absorbing states. If T_i be the time to system failure when it starts operation from state S_i then the reliability of the system is given by

$$R_i(t) = Pr[T_i>t]$$

Using the probabilistic arguments in view of reliability the following recursive relations can be easily obtained.

$$R_0(t) = Z_0(t) + q_{01}(t)©R_1(t) + q_{02}(t)©R_2(t)$$
$$R_1(t) = Z_1(t) + q_{10}(t)©R_0(t) + q_{13}(t)©R_3(t)$$
$$R_2(t) = Z_2(t) + q_{21}(t)©R_1(t)$$
$$R_3(t) = Z_3(t) + q^{(1)}{}_{30}(t)©R_0(t) + q^{(1)}{}_{33}(t)©R_3(t) + q_{35}(t)©R_5(t) \quad R_5(t) \quad R_5(t) = Z_5(t) + q_{50}(t)©R_0(t) + q_{53}(t)©R_3(t)$$
$$R_6(t) = Z_6(t) + q_{60}(t)©R_0(t) \qquad \text{….(47-52)}$$

where

$$Z_0(t) = e^{-\alpha t} \qquad Z_1(t) = e^{-(\beta+\delta)t}\,\overline{G}(t)$$

$$Z_2(t) = \overline{K}(t) \qquad Z_3(t) = \frac{\gamma}{\gamma-\beta-\delta}\,[\gamma e^{-(\beta+\delta)t} - (\beta+\delta)e^{-\gamma t})]\,\overline{G}(t)$$

$$Z_5(t) = e^{-(\alpha+\gamma)t} \qquad Z_6(t) = e^{-\alpha t}\,\overline{H}(t) \qquad \text{….(53-58)}$$

Taking Laplace transform of the above equations (47-52) and solving them for $R^*_0(s)$ by omitting the argument ‘s’ for brevity, we get

$$R^*_0(s) = N_1(s)/D_1(s) \qquad \text{….(59)}$$

where

$$N_1(s) = (1 - q^{*(1)}{}_{33} - q^*_{35}q^*_{53})(Z^*_0 + Z^*_1q^*_{01} + Z^*_1q^*_{02}q^*_{21}$$

$$+ Z^*_2q^*_{02}) + q^*_{13}(Z^*_3 + Z^*_5q^*_{35})(q^*_{01} + q^*_{02}q^*_{21}) \qquad \text{….(60)}$$

and

$$D_1(s) = (1 - q^{*(1)}{}_{33} - q^*_{35}q^*_{53})(1 - q^*_{01}q^*_{10} - q^*_{02}q^*_{21}q^*_{10})$$

$$- q^*_{13}(q^{*(1)}{}_{30} + q^*_{35}q^*_{50})(q^*_{01} + q^*_{02}q^*_{21}) \qquad \text{….(61)}$$

Now the reliability of the system can be obtained by taking inverse Laplace transform of (59) for well known failure time distributions. Also the Mean time to system failure (MTSF) when the system starts from the state S_0 can be obtained as

$$E(T_0) = \lim_{s\to 0} R^*_0(s) = N_1(0)/D_1(0) \qquad \text{….(62)}$$

Now,

$$Z^*_0(0) = \int_0^\infty e^{-\alpha t}\,dt = \frac{1}{\alpha} = \mu_0$$

$$Z^*_1(0) = \int_0^\infty e^{-(\beta+\delta)t}\,\overline{G}(t)dt$$
$$= \frac{1}{\beta+\delta}[1 - \widetilde{G}(\beta+\delta)] = \mu_1$$

$$Z^*_2(0) = {}_0\int^\infty \overline{K}(t)dt = m_1$$

$$Z^*_3(0) = \frac{\gamma}{\gamma-\beta-\delta} {}_0\int^\infty [\gamma e^{-(\beta+\delta)t} - (\beta+\delta)e^{-\gamma t})] \overline{G}(t)dt = L \text{ (say)}$$

$$Z_5(t) = {}_0\int^\infty e^{-(\alpha+\gamma)t}dt = \frac{1}{\alpha+\gamma} = \mu_5$$

$$Z_6(t) = {}_0\int^\infty e^{-\alpha t}\overline{H}(t)\, dt = \frac{1}{\alpha}[1 - \tilde{H}(\alpha)] = \mu_6 \qquad \text{....(63-68)}$$

Therefore,

$$N_1(0) = (1 - p^{(1)}_{33} - p_{35}p_{53})(\mu_0 + \mu_1 + m_1p_{02}) + p_{13}(L + \mu_5p_{35}) \qquad \text{....(69)}$$

and

$$D_1(0) = p_{14}(1 - p^{(1)}_{33} - p_{35}p_{53}) + p_{13}p^{(1,4)}_{36} \qquad \text{....(70)}$$

Hence, $E(T_0) = N_1/D_1$ ....(71)

where N_1 and D_1 are same as in (69) and (70).

7. Mean Up-Time of The System

Let $A_i(t)$ be the probability that starting from state S_i the system will remains up at epoch t. Using probabilistic arguments the following recursive relations can be easily obtained

$$A_0(t) = M_0(t) + q_{01}(t)©A_1(t) + q_{02}(t)©A_2(t)$$

$$A_1(t) = M_1(t) + q_{10}(t)©A_0(t) + q_{13}(t)©A_3(t) + q^{(4)}_{16}(t)©A_6(t)$$

$$A_2(t) = q_{21}(t)©A_1(t)$$

$$A_3(t) = M_3(t) + q^{(1)}_{30}(t)©A_0(t) + q^{(1)}_{33}(t)©A_3(t) + q_{35}(t)©A_5(t) + q^{(1,4)}_{36}(t)©A_6(t)$$

$$A_4(t) = q_{46}(t)©A_6(t)$$

$$A_5(t) = q_{50}(t)©A_0(t) + q_{53}(t)©A_3(t)$$

$$A_6(t) = M_6(t) + q_{60}(t)©A_0(t) + q_{64}(t)©A_4(t) \qquad \text{....(72-78)}$$

where

$$M_0(t) = e^{-\alpha t} \qquad M_1(t) = e^{-(\beta+\delta)t}\overline{G}(t)$$

$$M_3(t) = \frac{\gamma}{\gamma-\beta-\delta} e^{-(\beta+\delta+\gamma)t}\overline{G}(t) \qquad M_6(t) = e^{-\alpha t}\overline{H}(t) \qquad \text{....(79-82)}$$

Taking laplace transform of above equations (72-78) and solving for pointwise availability $A^*_0(s)$, by omitting the arguments ‘s’ for brevity, one gets

$$A^*_0(s) = \frac{N_2(s)}{D_2(s)} \qquad \text{....(83)}$$

where

$$N_2(s) = (1 - q^*_{46}q^*_{64})(1 - q^{*(1)}_{33} - q^*_{35}q^*_{53})[M^*_0 + M^*_1(q^*_{01} + q^*_{02}q^*_{21})] + q^*_{13}(q^*_{01} + q^*_{02}q^*_{21})$$
$$[(1 - q^*_{46}q^*_{64})q^*_{35}M^*_5 + q^{*(1,4)}_{36}M^*_6] + M^*_6q^{*(4)}_{16}(q^*_{01}$$

$$+ q^*_{02}q^*_{21})(1 - q^{*(1)}_{33} - q^*_{35}q^*_{53}) \quad ....(84)$$

and

$$D_2(s) = (1 - q^*_{46}q^*_{64})(1 - q^{*(1)}_{33} - q^*_{35}q^*_{53})[1 - q^*_{01}(q^*_{01} + q^*_{02}q^*_{21})] - q^*_{13}(q^*_{01} + q^*_{02}q^*_{21})[(1 - q^*_{46}q^*_{64}) .(1 - q^{*(1)}_{33} - q^*_{35}q^*_{53}) - q^{*(1,4)}_{36}(1 - q^*_{60} - q^*_{46}q^*_{64})] - (q^*_{01} + q^*_{02}q^*_{21})(1 - q^{*(1)}_{33} - q^*_{35}q^*_{53})q^{*(4)}_{16}q^*_{60}$$

....(85)

Thus,

$$D'_2(0) = p_{60}(1 - p^{(1)}_{33} - p_{35}p_{53})(\mu_0 + n_1 + m_1p_{02}) + [p_{13}p^{(1,4)}_{36} + p^{(4)}_{16}(1 - p^{(1)}_{33} - p_{35}p_{53})(\mu_6 + m_2p_{64}) \quad ....(86)$$

Also,

$$M^*_0(0) = {}_0\int^{\infty} e^{-\alpha t}dt = 1/\alpha = \mu_0$$

$$M^*_1(0) = {}_0\int^{\infty} e^{-(\beta+\delta)t}\,\overline{G}(t)\,dt = \mu_1$$

$$M^*_3(0) = \frac{\gamma}{\gamma-\beta-\delta}\,{}_0\int^{\infty}(e^{-(\beta+\delta)t} - e^{-\gamma t}).\,\overline{G}(t)dt = \frac{(\mu_1-\mu_3)\gamma}{(\gamma-\beta-\delta)} = L_1 \text{ (say)}$$

$$M^*_6(0) = {}_0\int^{\infty} e^{-\alpha t}\,\overline{H}(t)\,dt = \mu_6 \quad ....(87\text{-}90)$$

and then

$$N_2(0) = (1 - p^{(1)}_{33} - p_{35}p_{53})[p_{60}(\mu_0 + \mu_1) + p^{(4)}_{16}\mu_6] + p_{13}[p^{(1,4)}_{36}\mu_6 + p_{60}(p_{35}\mu_5 + L_1)] \quad ....(91)$$

Therefore,

$$A_0 = \lim_{t\to\infty} U_0(t) = \lim_{s\to 0} s.U^*_0(s)$$

$$= \lim_{s\to 0} N_2(s)/D_2(s) = N_2/D_2 \quad ....(92)$$

Where N_2 and D_2 are same as in equations (92) and (86) respectively.

8. Mean Down-Time Of The System

Let $B_i(t)$ be the probability that starting from regenerative down state S_j the system will remains down at epoch t. Using probabilistic arguments the following recursive relations among Bi(t)'s can be easily obtained

$$B_0(t) = q_{01}(t)©B_1(t) + q_{02}(t)©B_2(t)$$

$$B_1(t) = q_{10}(t)©B_0(t) + q_{13}(t)©B_3(t) + q^{(4)}_{16}(t)©B_6(t)$$

$$B_2(t) = M_2(t) + q_{21}(t)©B_1(t)$$

$$B_3(t) = M_3(t) + q_{30}(t)©B_0(t) + q^{(1)}_{33}(t)©B_3(t) + q_{35}(t)©B_5(t) + q^{(1,4)}_{36}(t)©B_6(t)$$

$$B_4(t) = q_{46}(t)©B_6(t)$$

$$B_5(t) = q_{50}(t)©B_0(t) + q_{53}(t)©B_3(t)$$

$$B_6(t) = q_{60}(t)©B_0(t) + q_{64}(t)©B_4(t) \quad ....(93\text{-}99)$$

where

$$M_2(t) = \overline{K}(t) \qquad M_3(t) = e^{-\gamma t}\,\overline{G}(t) \quad ....(100\text{-}101)$$

Taking Laplace transform of above equations (93-99) and solving them for $B^*_0(s)$, by omitting the arguments 's' for brevity, one gets

$$B^*_0(s) = \frac{N_3(s)}{D_3(s)} \quad ....(102)$$

where

$$N_3(s) = (1 - q^*_{46}q^*_{64})(1 - q^{*(1)}_{33} - q^*_{35}q^*_{53})M^*_2q^*_{02} + q^*_{13}(q^*_{01} + q^*_{02}q^*_{21})(1 - q^*_{46}q^*_{64})M^*_3 \quad ....(103)$$

and $D_3(s)$ is same as $D_2(s)$ in equation (85).

Now,

$$M^*_2(0) = {}_0\int^{\infty} \overline{K}(t)\,dt = \mu_2 = m_1$$

$$M^*_3(0) = {}_0\int^{\infty} e^{-\gamma t}\,\overline{G}(t)\,dt = \mu_3 \quad ....(104\text{-}105)$$

Then,

$$N_3(0) = p_{60}[(1 - p^{(1)}_{33} - p_{35}p_{53})p_{02}m_1 + p_{13}\mu_3] \quad ....(106)$$

Therefore,

$$B_0 = \lim_{t\to\infty} A_0(t) = \lim_{s\to 0} s.B^*_0(s)$$

$$= \lim_{s\to 0} N_3(s)/D_3(s) = N_3(0)/D'_3(0) = N_3/D_3 \quad ....(107)$$

where N_3 is same as in (106) and D_3 is as D_2 in (86).

9. Expected Number of Repairs of Failed Unit

Let we define, $V_i(t)$ as the expected number of repairs by the repairman in (0,t] given that the system initially started from regenerative state S_i at t=0. Then following recurrence relations among $V_i(t)$'s can be obtained as;

$$V_0(t) = Q_{01}(t)\$V_1(t) + Q_{02}(t)\$V_2(t)$$

$$V_1(t) = Q_{10}(t)\$[1 + V_0(t)] + Q_{13}(t)\$V_3(t) + Q^{(4)}_{16}(t)\$[1+V_6(t)]$$

$$V_2(t) = Q_{21}(t)\$V_1(t)$$

$$V_3(t) = Q^{(1)}_{30}(t)\$[1 + V_0(t)] + Q^{(1)}_{33}(t)\$V_3(t) + Q_{35}(t)\$[1 + V_5(t)] + Q^{(1,4)}_{36}(t)\$[1+ V_6(t)]$$

$$V_4(t) = Q_{46}(t)\$[1 + V_6(t)]$$

$$V_5(t) = Q_{50}(t)\$V_0(t) + Q_{53}(t)\$V_3(t)$$

$$V_6(t) = Q_{60}(t)\$[1 + V_0(t)] + Q_{64}(t)\$V_4(t) \quad ....(108\text{-}114)$$

Taking laplace stieltjes transform of the above equations (108-114) and the solution of $\tilde{V}_0(s)$ may be expressed by omitting the argument 's' for brevity as

$$\tilde{V}_0(s) = N_4(s)/D_4(s) \quad ....(115)$$

Where

$$N_4(s) = (1 - \tilde{Q}_{46}\tilde{Q}_{64})(1 - \tilde{Q}^{(1)}_{33} - \tilde{Q}_{35}\tilde{Q}_{53})(\tilde{Q}_{10} + \tilde{Q}^{(4)}_{16})(\tilde{Q}_{01}$$

$$+ \tilde{Q}_{02}\tilde{Q}_{21}) + \tilde{Q}^{(4)}_{16}(\tilde{Q}_{46}\tilde{Q}_{64} + \tilde{Q}_{60})(1 - \tilde{Q}^{(1)}_{33}$$
$$- \tilde{Q}_{35}\tilde{Q}_{53})(\tilde{Q}_{01} + \tilde{Q}_{02}\tilde{Q}_{21}) + \tilde{Q}_{01}\tilde{Q}_{13}[(\tilde{Q}^{(1)}_{30} + \tilde{Q}_{35}$$
$$+ \tilde{Q}^{(1,4)}_{36})(1 - \tilde{Q}_{46}\tilde{Q}_{64}) + \tilde{Q}^{(1,4)}_{36}(\tilde{Q}_{46}\tilde{Q}_{64} + \tilde{Q}_{60})] \quad \text{....(116)}$$

And

$$D_4(s) = (1 - \tilde{Q}_{46}\tilde{Q}_{64})(1 - \tilde{Q}^{(1)}_{33} - \tilde{Q}_{35}\tilde{Q}_{53})[1 - \tilde{Q}_{10}(\tilde{Q}_{01}$$
$$+ \tilde{Q}_{02}\tilde{Q}_{21})] - \tilde{Q}_{13}(\tilde{Q}_{01} + \tilde{Q}_{02}\tilde{Q}_{21})[(1 - \tilde{Q}_{46}\tilde{Q}_{64})(1 - \tilde{Q}^{(1)}_{33}$$
$$- \tilde{Q}_{35}\tilde{Q}_{53}) - \tilde{Q}^{(1,4)}_{36}(1 - \tilde{Q}_{60} - \tilde{Q}_{46}\tilde{Q}_{64})] - (\tilde{Q}_{01}$$
$$+ \tilde{Q}_{02}\tilde{Q}_{21})(1 - \tilde{Q}^{(1)}_{33} - \tilde{Q}_{35}\tilde{Q}_{53})\tilde{Q}^{(4)}_{16}\tilde{Q}_{60} \quad \text{....(117)}$$

Now,

$$N_4(0) = (1 - p^{(1)}_{33} - p_{35}p_{53})(p_{60}(1 - p_{13}) + p^{(4)}_{16})$$
$$+ p_{01}p_{13}[p_{60}(1 - p^{(1)}_{33}) + p^{(1,4)}_{36}] \quad \text{....(118)}$$

Therefore, in steady state the expected number of repairs for failed unit is

$$V_0 = \lim_{t \to \infty} [V_0(t)/t] = \lim_{s \to 0} s\,\tilde{V}_0(s) = N_4/D_4 \quad \text{....(119)}$$

where N_4 is same as (118) and D_4 is same as D_2 in (86).

10. Expected Number of Repairs of Automatic Switch

Let we define, $V'_i(t)$ as the expected number of repairs for automatic switch by the repairman in (0,t] given that the system initially started from regenerative state S_i at t=0. Then following recurrence relations among $V'_i(t)$'s can be obtained as;

$$V'_0(t) = Q_{01}(t)\$V'_1(t) + Q_{02}(t)\$V'_2(t)$$
$$V'_1(t) = Q_{10}(t)\$V'_0(t) + Q_{13}(t)\$V'_3(t) + Q^{(4)}_{16}(t)\$V'_6(t)$$
$$V'_2(t) = Q_{21}(t)\$V'_1(t)$$
$$V'_3(t) = Q^{(1)}_{30}(t)\$V'_0(t) + Q^{(1)}_{33}(t)\$V'_3(t) + Q_{35}(t)\$V'_5(t)$$
$$+ Q^{(1,4)}_{36}(t)\$V'_6(t)$$
$$V'_4(t) = Q_{46}(t)\$V'_6(t)$$
$$V'_5(t) = Q_{50}(t)\$V'_0(t) + Q_{53}(t)\$V'_3(t)$$
$$V'_6(t) = Q_{60}(t)\$V'_0(t) + Q_{64}(t)\$V'_4(t) \quad \text{....(120-126)}$$

Taking laplace stieltjes transform of the above equations (120-126) and the solution of $\tilde{V}_0(s)$ may be expressed as by omitting the argument 's' for brevity is

$$\tilde{V}_0(s) = N_5(s)/D_5(s) \qquad ....(127)$$

where

$$N_5(s) = (1 - \tilde{Q}_{46}\tilde{Q}_{64})(1 - \tilde{Q}^{(1)}_{33} - \tilde{Q}_{35}\tilde{Q}_{53})\tilde{Q}_{02}\tilde{Q}_{21} \qquad ....(128)$$

and $D_5(s)$ is same as in $D_4(s)$ in (117).

Now,

$$N_5(0) = p_{60}p_{02}(1 - p^{(1)}_{33} - p_{35}p_{53}) \qquad ....(129)$$

Therefore, in steady state the expected number of repairs of automatic switch is

$$V_0 = \lim_{t \to \infty} [V_0(t)/t] = \lim_{s \to 0} s\,\tilde{V}_0(s) = N_5/D_5 \qquad ....(130)$$

where N_5 is same as (129) and D_4 is same as D_2 in (86).

11. Conclusion

The paper provides the reliability characteristics such as MTSF, Availability, Busy Period Analysis, etc. by using regenerative point technique with Markov renewal process, of a fuel supply system in an automobile engine of a four wheeler which is having both the option of fuel i.e. PETROL and CNG.

12. References

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