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Comparison of the Design of Flexural Reinforced Concrete Elements According to Albanian Normative

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***Abstract*: *In this work, the authors present the findings obtained from the analysis of calculating the bearing capacity of reinforced concrete elements under the bending moment. The chose element, a reinforced concrete beam, is designed according to Albanian Normative with two methods, allowable stress design method and limit state design method. Furthermore, is made a comparison between the analysis results. In the beginning a presentation is made with the theoretical solution of the problem and after that the comparison is based on numerical solution. The conclusions are followed from the recommendations given in the end of this work.*

Keywords: *allowable stress design method, limit state design method, design of flexural elements, Albanian Normative*

1. Introduction

The design of reinforced concrete structures, in different time periods, is performed in accordance with approved technical normative, using three methods [1]

- allowable stress design method or classic method
- rupture method
- limit state design method

In years ’60 – ’70 of the last century, in Albania, the allowable stress design method was used for the design of motorway or railway bridges and also for the design of hydro technical structures where the cracking where not allowed. The rupture method was used for the design of industrial and civil buildings. The limit state design method was used without restrictions for reinforced concrete structures and also for the prestressed concrete structures. This method is the one that is used actually in our country. The comparison of the results of the calculation of the bearing bending moment of the same element according to the two methods, shows the usage advantage of the limit state design method against allowable stress design method.

2. Design Methods of Reinforced Concrete Elements According To Albanian Normative.

2.1. Symbols According the Two Methods

The symbols between {…} belong to the allowable stress design method. Only the symbols that differ are shown. The same symbols are not shown.

A_s – reinforcing steel area on the tensile zone; $\{F_a\}$

A_{sc} – reinforcing steel area on the compression zone; $\{F'_a\}$

R_s – design strength of reinforcing steel A_s; $\{R_a\}$

R_{sc} – design strength of reinforcing steel A_{sc}; Usually $R_s = R_{sc}$ because the same steel is used; {R'_a; usually $R_a = R'_a$}

σ_s – reinforcing steel stresses on the tensile zone; {σ_a}

σ'_s – reinforcing steel stresses on the compression zone; {σ'_a}

E_s – reinforcing steel modulus of elasticity; {E_a}

2.2. Allowable Stress Design Method

In Albania this is the first and the oldest method and for this reason is also known as the classic method. This method is based on the elastic phase of work of concrete and the reinforcement, so it does not use the plastic capability of materials. The hypothesis where this method is based:

- accept and use formulas of the construction science
- Accept the Bernul's hypothesis, which do not accept the deformation of cross section but only the rotation or movement of the cross section.
- Concrete in tensile zone make no conctribution. All the stresses in the tensile zone are hold only by the reinforcing steel.
- Stresses chart in the compression zone is of a triangular linear shape. See Figure 2.1.
- Convert the reinforced concrete element (concrete + reinforcing steel) in a only concrete element. For this reason, the reinforcing steel area is multiplied by number 'n', where $n = E_a/E_b$. E_a and E_b are respectively reinforcing steel modulus of elasticity and concrete modulus of elasticity.
- Consider the reinforced concrete as a homogenous and isotropic material

Based on the above hypothesis, the stress distribution on the reinforced concrete element with reinforcing steel in tensile and compressive zone under the bending moment, with a cross section symmetric against the vertical axis, is given in Figure 2.1.

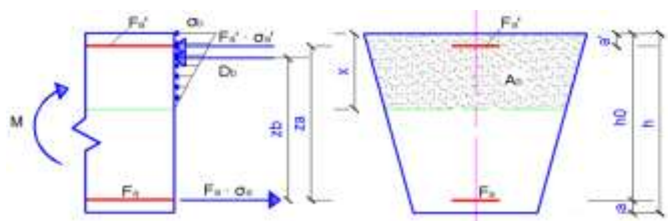


Fig. 2.1: Stress distribution according the the allowable stress design method

In Figure 2.2 is showed the stress distribution in an element with rectangualr cross section.

σ_b – constrains of concrete in the compression zone

D_b – equivalent force in the compression zone

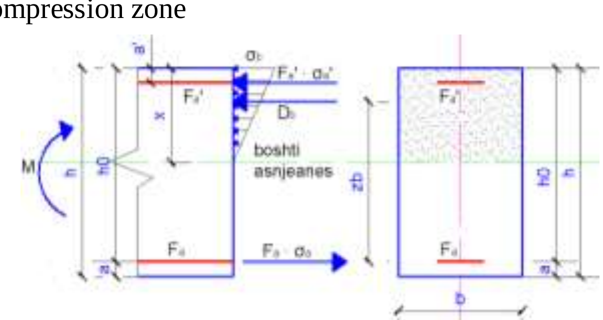


Fig. 2.2: Stress distribution in rectangular cross section according to the allowable stress design method.

Given the hypothesis where this method is based [1]:

$$F_{ek} = F_b + n \cdot F_a + n \cdot F_a' \quad (2.1)$$

$$F_b = b \cdot h \quad (2.2)$$

Static moment against a random axis:

$$S_{ek} = S_b + n \cdot S_a + n \cdot S_a' \tag{2.3}$$

Moment of inertia against a random axis:

$$I_{ek} = I_b + n \cdot I_a + n \cdot I_a' \tag{2.4}$$

The basic equation that shows the essence of this method is:

$$\text{Action} \leq \text{Bearing capacity} \tag{2.5}$$

As action will be the bending moment, shear force, etc. The respective bearing capacity is also a bending moment, shear force, etc. It depends from the allowable stresses in concrete $[\sigma_b]$, from the allowable stresses in the tensile reinforcing $[\sigma_a]$, from the allowable stresses in the compressed reinforcing $[\sigma_a']$, from element's cross section dimensions (b and h), from the area of tensile reinforcing (F_a) and the compression reinforcing (F_a').

In case of bending the equation (2.5) is written:

$$\text{Acting bending moment} \leq \text{Bearing bending moment} \tag{2.6}$$

Controls that are carried out:

$$\sigma_b = M \cdot x / I_{ek} \leq [\sigma_b] \tag{2.7}$$

$$\sigma_a = n \cdot \sigma_b = n \cdot M \cdot (h_0 - x) / I_{ek} \leq [\sigma_a] \tag{2.8}$$

If condition (2.7) is fulfilled, than condition (2.9) also is always fulfilled:

$$\sigma_a' = n \cdot M \cdot (x - a') / I_{ek} \leq [\sigma_a'] \tag{2.9}$$

To determine the position of the neutral axis we start from the fact that the static moment against it should be equal to zero. In view of figure 2.2 we can write:

$$0.5 \cdot b \cdot x^2 + n \cdot F_a' \cdot (x - a') - n \cdot F_a \cdot (h_0 - x) = 0 \tag{2.10}$$

From where:

$$x = \frac{n \cdot (F_a + F_a')}{b} \cdot \left[-1 + \sqrt{1 + \frac{2 \cdot b \cdot (F_a \cdot h_0 + F_a' \cdot a')}{n \cdot (F_a + F_a')^2}} \right] \tag{2.11}$$

Moment of inertia:

$$I_{ek} = \frac{b \cdot x^3}{3} + n \cdot F_a \cdot (h_0 - x)^2 + n \cdot F_a' \cdot (x - a')^2 \tag{2.12}$$

2.3. Limit State Design Method

According to Albanian normative [7] we have three limit states:

- Ultimate limit state
- Serviceability limit state according to deformation
- Serviceability limit state according to cracking

Ultimate limit state calculations have to be always performed. Calculations for other two limit states may be performed in special occasions, where is seen reasonable. Hypotheses used are [7]:

- Planar cross sections under the bending moment rotate, but remain planar.
- Concrete works only in the compression zone. Concrete stress chart in compression zone is accepted constant. Stress values are equal to the concrete compression strength, R_b.
- There is no contribution from the concrete in bearing the stresses in tensile zone.
- Only the reinforcing steel work in tensile zone. Stresses in reinforcing steel are R_s.
- If there is reinforcing steel in compression zone, the stresses in it are R_{sc}.
- Strains in reinforcing steel and in the concrete in its vicinity are the same.
- Ultimate relative strain in compression for concrete is 0.2%.

Based on the above hypothesis, the stress distribution on the reinforced concrete element with reinforcing steel in tensile and compressive zone under the bending moment, with a cross section symmetric against the vertical axis, is given in Figure 3.1.

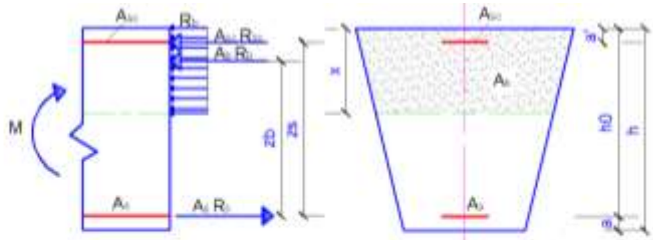


Fig. 3.1Stress distribution according limit state design method

The determination of the position of the neutral axis is made by following equations:

$$x_y = \xi_y \cdot h_0 \quad (3.1)$$

$$\xi_y = \frac{\omega}{2 - \frac{\omega}{1.1}} \quad (3.2)$$

For heavy concrete: $\omega = 0.85 - 0.008 \cdot R_b$ (3.3)

For light concrete: $\omega = 0.80 - 0.008 \cdot R_b$ (3.4)

where R_b in N/mm^2.

If $x \leq x_y$, then there is no need for A_{sc}, so $A_{sc} = 0$. If $x > x_y$, then A_{sc} is needed, so $A_{sc} \neq 0$. In this case there is a risk for compression zone failure. To avoid this case A_{sc} is used.

Calculation in case where there is no need for A_{sc}, ($x \leq x_y$, $A_{sc} = 0$).

According to figure 3.1 we can write two equilibrium equations.

$$M = A_b \cdot R_b \cdot z_b \quad (3.5)$$

$$A_s \cdot R_s - A_b \cdot R_b = 0 \quad (3.6)$$

Calculation in case where there is no need for A_{sc}, ($x > x_y$, $A_{sc} \neq 0$).

According to figure 3.1 we can write two equilibrium equations.

$$M = A_b \cdot R_b \cdot z_b + A_{sc} \cdot R_{sc} \cdot z_s \quad (3.7)$$

$$A_s \cdot R_s - A_b \cdot R_b - A_{sc} \cdot R_{sc} = 0 \quad (3.8)$$

Design of reinforced concrete elements with rectangular cross section ($x \leq x_y$, $A_{sc} = 0$). Stress distribution is shown on figure 3.2.

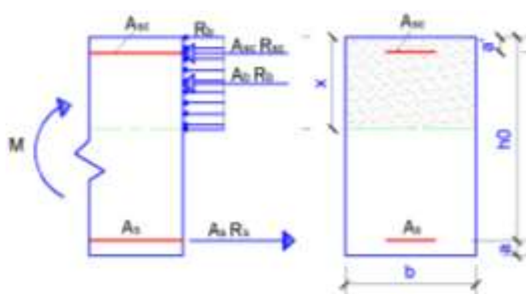


Fig. 3.2: Stress distribution in rectangular cross section according to the limit state design method

From (3.5) we have:

$$M = b \cdot x \cdot R_b \cdot (h_0 - 0.5 \cdot x) \quad (3.9)$$

Lets write $\xi = x / h_0$. Substituting on (3.9):

$$M = b \cdot h^2_0 \cdot R_b \cdot A_0 \quad (3.10)$$

$$A_0 = \xi \cdot (1 - 0.5 \cdot \xi) \quad (3.11)$$

From equation (3.10) find A_0:

$$A_0 = \frac{M}{b \cdot h_0^2 \cdot R_b} \quad (3.12)$$

$$\xi = 1 - (1 - 2 \cdot A_0)^{1/2} \quad (3.13)$$

$$x = \xi \cdot h_0 \quad (3.14)$$

If x calculated from equation (3.14) $\leq x_y$, determined from equation (3.2), then from equation (3.6) we can find A_s:

$$A_s = \xi \cdot b \cdot h_0 \cdot R_b / R_s \quad (3.15)$$

Design of reinforced concrete element with rectangular cross section ($x > x_y$, $A_{sc} \neq 0$) Let see again figure 3.2. To maximally utilize the compressed zone of concrete we accept $x = x_y = \xi_y \cdot h_0$. Equation (3.7) can be writte:

$$M = b \cdot x_y \cdot R_b \cdot (h_0 - 0.5 \cdot x_y) + A_{sc} \cdot R_{sc} \cdot (h_0 - a') \quad (3.16)$$

After some transformations we have:

$$A_{sc} = \frac{M - A_{0y} \cdot b \cdot h_0^2 \cdot R_b}{R_{sc} \cdot (h_0 - a')} \quad (3.17)$$

$$A_{oy} = \xi_y \cdot (1 - 0.5 \cdot \xi_y) \quad (3.18)$$

From (3.8) we have:

$$A_s = A_{sc} + \xi_y \cdot b \cdot h_0 \cdot R_b / R_s \quad (3.19)$$

2.4. Results of the Calculations of Flexural Reinforced Concrete Elements

Numerical example 1

A beam under flexural action is analyzed. Rectangular cross section with b = 30cm and h = 50cm, a = a' = 3.5cm, h_0 = h–a = 50–3.5 = 46.5cm. According to allowable stress design method concrete is of a mark M 300 (cubic resistance), $[\sigma_b]$ = 135daN/cm^2; steel Ç.5, $[\sigma_a]$ = 1600daN/cm^2; n = E_a / E_b = 10. According to limit state design method the concrete is of class B30 (cubic resistance), R_b = 160daN/cm^2, E_b = 306000daN/cm^2; steel Ç.5, R_s = 2400daN/cm^2, E_s = $2 \cdot 10^6$daN/cm^2; n = E_s / E_b = 6.535. A_{sc} (F_a') = 0cm^2; A_s (F_a) = 4φ16 = 4·2.01 = 8.04cm^2.

Determine, with the two methods, the flexural strength of the beam.

a) Allowable stress design method.

With help of equation (2.11) determine x = 13.33cm.

With help of equation (2.12) determine I_{ek} = 112128cm^4.

With help of equation (2.7) determine the bearing moment M = 1135405daN·cm.

With help of equation (2.8) determine the bearing moment M = 540900daN·cm.

Finally the bearing moment is the smallest between those that are determined from (2.7) and (2.8). In this case 540900 daN·cm

b) Limit state design method.

With help of equation (3.6) determine $x = (A_s \cdot R_s) / (b \cdot R_b)$ = 4.02cm.

With help of equation (3.9) determine the bearing moment M = 858479daN·cm.

With limit state design method, the bearing moment results 58% greater.

Numerical example 2

A beam under flexural action is analyzed. Rectangular cross section with b = 30cm and h = 50cm, a = a' = 3.5cm, h_0 = h–a = 50–3.5 = 46.5cm. According to allowable stress design method concrete is of a mark M 300 (cubic resistance), $[\sigma_b]$ = 135daN/cm^2; steel Ç.5, $[\sigma_a]$ = 1600daN/cm^2; n = E_a / E_b = 10. According to limit state design method the concrete is of class B30 (cubic resistance), R_b = 160daN/cm^2, E_b = 306000daN/cm^2; steel Ç.5, R_s = 2400daN/cm^2, E_s = $2 \cdot 10^6$daN/cm^2; n = E_s / E_b = 6.535. A_{sc} (F_a') = 2φ10 = 1.57cm^2; A_s (F_a) = 4φ16 = 4·2.01 = 8.04cm^2.

Determine, with the two methods, the flexural strength of the beam.

a) Allowable stress design method.

With help of equation (2.11) determine x = 12.56cm.

With help of equation (2.12) determine I_{ek} = 115728cm^4.

With help of equation (2.7) determine the bearing moment M = 460830daN·cm.

With help of equation (2.8) determine the bearing moment M = 545510daN·cm.

Finally the bearing moment is the smallest between those that are determined from (2.7) and (2.8). In this case 460830 daN·cm.

b) Limit state design method.

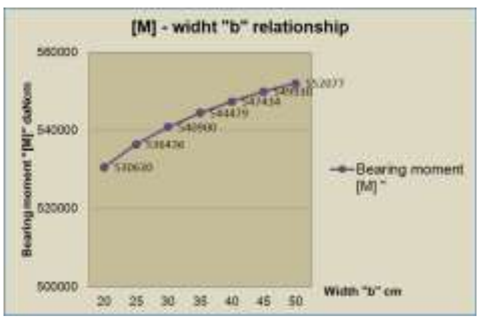
With help of equation (3.8) determine $x = (A_s \cdot R_s - A_{sc} \cdot R_{sc}) / (b \cdot R_b)$ = 3.23cm.

With help of equation (3.7) determine the bearing moment M = 858959daN·cm.

With limit state design method, the bearing moment results 86% greater.

2.5. Calculations, Analysis, Results

To make possible the comparison of the calculation results, the elements are considered in the same conditions. The same class of concrete and steel is accepted, the same quantity of reinforcing steel (compressed and tensile), the same cross section dimensions. Graphically is showed the connection between the bearing bending moment and other factors as: width of cross section, height of cross section, concrete allowable stresses, steel allowable stresses, E_S/E_b ratio, quantity of tensile reinforcement, quantity of compressed reinforcement.



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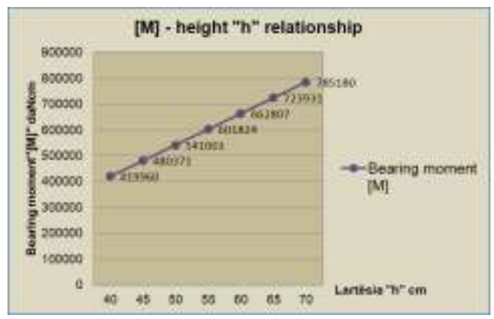

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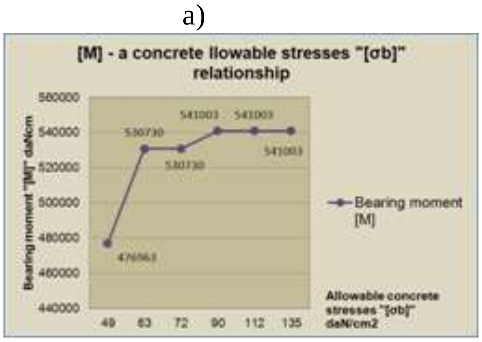

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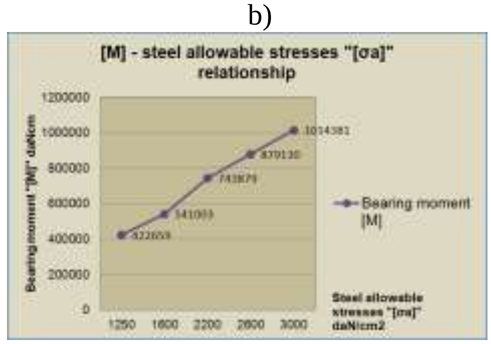

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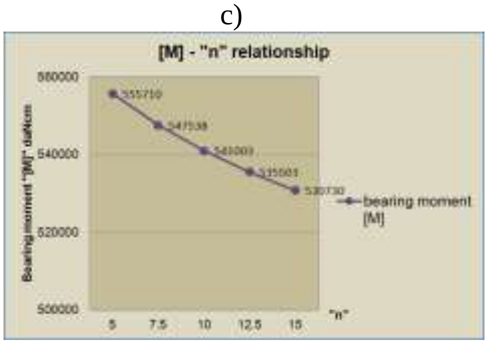

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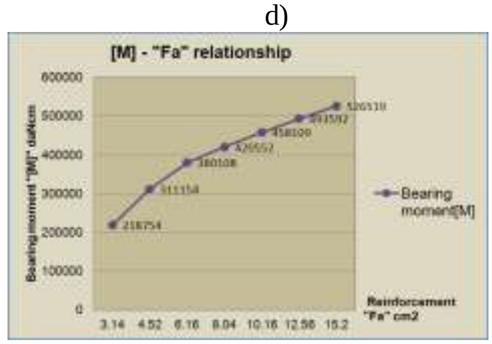

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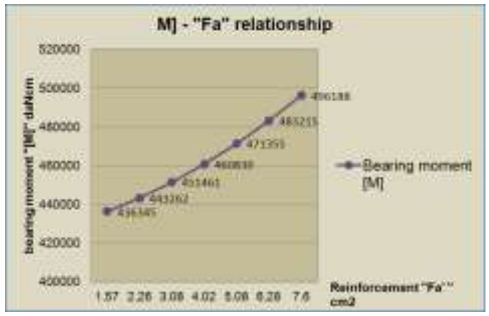
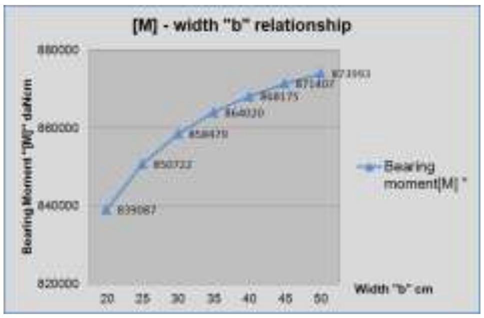
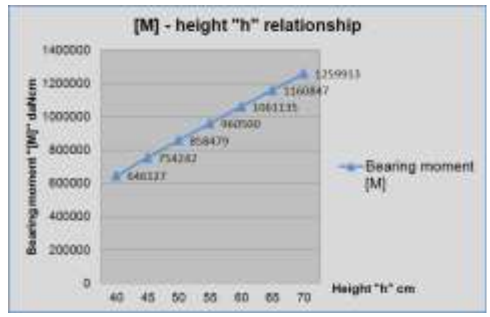


g)

Fig. 5.2: Relationship between bearing bending moment and different factors, according to allowable stress design method.



a)



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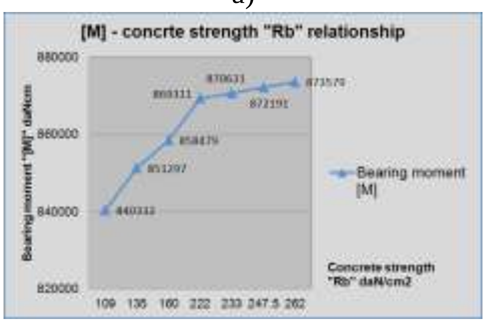

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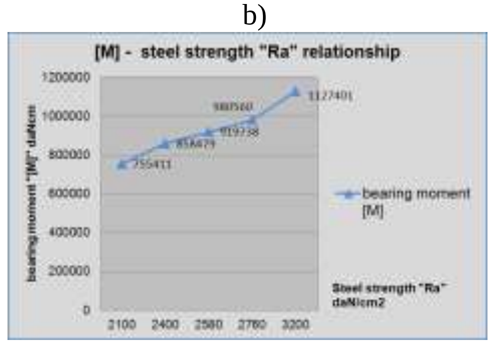

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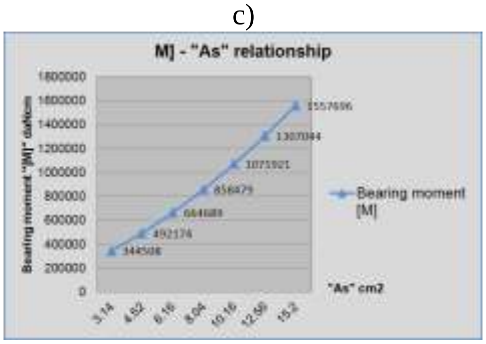

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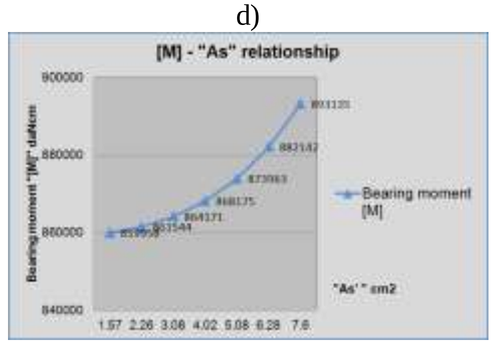

f)

Fig. 5.3: Relationship between bearing bending moment and different factors, according to limit state design method.

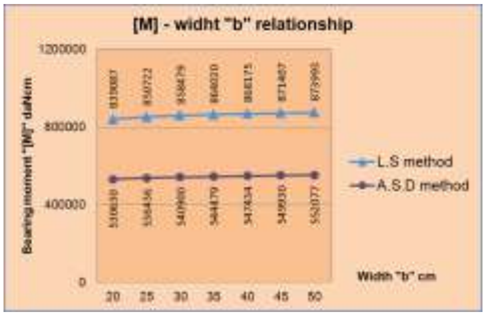

Fig. 5.4: Relationship between bearing bending moment and cross section width, method comparison.

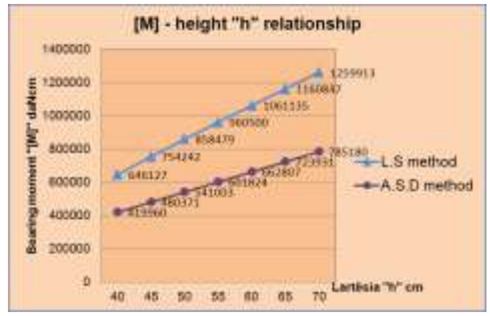

Fig. 5.5: Relationship between bearing bending moment and cross section height, comparison method

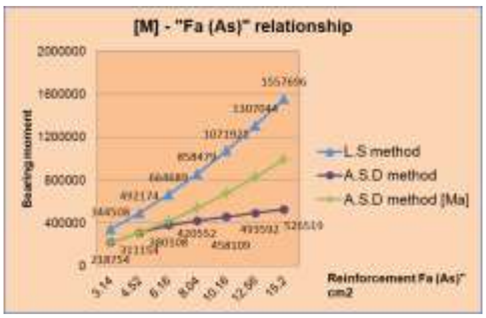

Fig. 5.6: between bearing bending moment and tensile steel area, method comparison

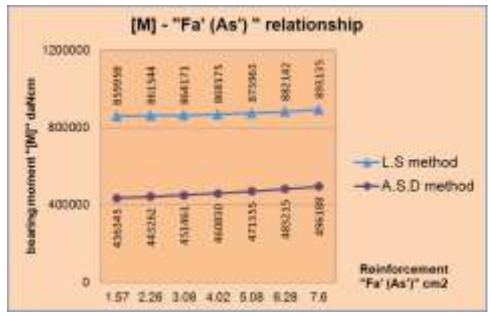

Fig. 5.7: Relationship between bearing bending moment and compressed steel area, method comparison

3. Conclusions

- In all the cases the bearing moment strength, calculated with the limit state method (L.S) is higher compared to that calculated with the allowable strength method (A.S.D). This is an expected result knowing that the L.S use better the contribution of concrete and of the reinforcing steel in the elastic phase. See all the figures, specifically figures 6.3 to 6.6.
- With the increasing of cross section width ‘b’ also the bearing moment is increased. Si figure 6.3. The diagrams are almost parallel. The impact of ‘b’ is equal to both the methods.
- Impact of “b” in the bearing moment is small. A 2.5 time increase of width bring a 4% increase of the bearing moment, calculated with both the methods.
- With the increasing of cross section height ‘h’ also the bearing moment is increased. See figure 6.4. Impact of “h” is more significant in the L.S method; the respective diagram is more inclined.
- Impact of “h” in the bearing moment is greater. An increase of 1.75 time of height brings a 95% increase in the bearing moment according to L.S and a 87% according to A.S.D.
- With the increasing of tensile reinforcing area “F_a (A_s)” also the bearing moment is increased. See figure 6.5. Ultimate bearing moment according to A.S.D depend from allowable stresses in concrete. Green diagram correspond to the bearing moment connected with the allowable stresses in the reinforcing steel. Comparing this diagram with that of L.S method, result that the impact of the tensile reinforcement is more significant in the L.S method. The respective diagram is more inclined.
- The impact of the quantity of tensile reinforcement is high. An increase of 4.84 time the area of reinforcement bring approximately a 4.5 time increase according to L.S and 4.3 time according to A.S.D of the bearing moment.
- With the increasing of compressed reinforcement “F_a’ (A_s’)” also the bearing moment is increased. See figure 6.6. The diagram is almost parallel. The impact of “F_a’ (A_s’)” is almost equal for both the methods.

- The impact of the quantity of compressed reinforcement is high. An increase of 4.84 time the area of reinforcement bring approximately a 4.5 time increase according to L.S and 4.3 time according to A.S.D of the bearing moment.
- Bearing moment calculated with L.S method does not depend from “n”.
- The dependence of the bearing moment from the concrete allowable stress, calculated with A.S.D method, is complicated because the moment depends also from ‘n’ See figure 6.1c. According to Albanian normative n=10 for concrete mark < 200 and n=15 for concrete mark ≥ 200. Last three dots in figure 6.1c diagram correspond to the first three dots of the diagram in figure 6.2c. Is seen that also in this case the values of bearing moments calculated according to L.S method are higher.
- As conclusion we can say that the transition from the allowable stress design method to the ultimate state method, for design from bending moment, according to Albanian normative, for the flexural reinforced concrete elements, brings a decrease of the quantity of reinforcing steel and concrete, decreasing the cost.

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