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Mass and Stiffness Updating Based on Normal Response Function Method

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***Abstract*:** *A method of model updating to estimate mass and stiffness matrices for damped system is presented, which is the extension of the available cross-model cross-mode method. The concept of normal FRF is employed for extracting the FRF of undamped system from the measured complex FRF. Moreover, a simple algorithm for the case of incomplete data is also presented. A numerical example is demonstrated the effectiveness of the proposed method.*

Keywords: *Cross-model cross-mode method, Incomplete data, Model updating, Normal frequency response function*

1. Introduction

Updating of the structural dynamic finite element (FE) models has been an active area of research for the last two decades. According to the literature review, model updating methods can be broadly classified into three major groups: direct matrix methods [1-4], parametric updating methods [5-8] and cross-model cross-mode (CMCM) method [9-11]. Direct matrix methods are generally of non-iterative methods, which is to modify the mass and stiffness matrices directly. Although these methods can reproduce measured modal data exactly, they can't preserve structural connectivity and the updated results are not physically meaningful. Parametric updating methods, which are the iterative methods, taking the structural design parameters as updating parameters, seek to find their correction factors. These methods can maintain physical connectivity well and they are very easy to apply into the business engineering software. The CMCM method is recently developed by Hu et al. [9] for the simultaneous updating of the mass and stiffness matrices, which is a non-iterative method. The CMCM method forms a set of linear simultaneous equations, in which each equation is formulated based on the two same/different modes (mode shapes and modal frequencies) associated with the FE and experimental models. Different from parametric updating methods, the CMCM method allows choosing more updating parameters which don't depend on the sensitivity analysis, and the measured and analytical modes don't require to be paired. In [11], the CMCM method is extended based on FRFs for undamped system (called the CMCF method), which can avoid the influence of modal analysis errors on the updating process. However, updating of mass and stiffness based on complex FRFs is not theoretically correct due to the influence of damping. This note addresses this difficulty and proposes an improved version using normal FRFs for damped system, and also extends it for the case of incomplete data.

2. Mass and Stiffness Updating

2.1. Updating method

The governing equations of motion of a multi degree of freedom (DOF) structure with both viscous and structural damping can be written in matrix form as,

Where $\mathbf{M}$, $\mathbf{K}$, $\mathbf{C}$ and $\mathbf{D}$ are mass, stiffness, viscous damping and structural damping matrices respectively. $\boldsymbol{x}(t)$ and $\boldsymbol{f}(t)$ are the vectors of displacements and external forces.

Transforming Eq. (1) into frequency domain, the following matrices can be obtained as

$$\mathbf{Z}(\omega)^C = \mathbf{K} - \omega^2\mathbf{M} + i(\mathbf{D} + \omega\mathbf{C}) \tag{2}$$

$$\mathbf{H}(\omega)^C = (\mathbf{Z}(\omega)^C)^{-1} \tag{3}$$

$$\mathbf{Z}(\omega)^N = \mathbf{K} - \omega^2\mathbf{M} \tag{4}$$

$$\mathbf{H}(\omega)^N = (\mathbf{Z}(\omega)^N)^{-1} \tag{5}$$

where $\mathbf{Z}(\omega)^C$ and $\mathbf{Z}(\omega)^N$ are the complex dynamic stiffness matrix (DSM) and normal DSM respectively; $\mathbf{H}(\omega)^C$ and $\mathbf{H}(\omega)^N$ are the complex frequency response function (FRF) matrix and normal FRF matrix, respectively. The complex FRFs have both real and imaginary parts, which can be obtained by direct measurement. The normal FRFs which only have real part are FRFs for an undamped structure, which cannot be directly measured for a practical structure. However, they can be computed from the knowledge of complex FRFs (Chen et al. [12]), which is formulated as

$$\mathbf{H}(\omega)^N = \mathbf{H}(\omega)_R^C + \mathbf{H}(\omega)_I^C(\mathbf{H}(\omega)_R^C)^{-1}\mathbf{H}(\omega)_I^C \tag{6}$$

where $\mathbf{H}(\omega)_R^C$, $\mathbf{H}(\omega)_I^C$ are the real and imaginary part of complex FRF matrix $\mathbf{H}(\omega)^C$.

Now, the following identities relating the normal DSM and normal FRF matrix can be written for the analytical FE model and experiment model respectively as

$$\mathbf{Z}(\omega_i)_A^N\mathbf{H}(\omega_i)_A^N = \mathbf{I} \quad or \quad (\mathbf{K}_A - \omega_i^2\mathbf{M}_A)\mathbf{H}(\omega_i)_A^N = \mathbf{I} \tag{7}$$

$$\mathbf{Z}(\omega_j)_X^N\mathbf{H}(\omega_j)_X^N = \mathbf{I} \quad or \quad (\mathbf{K}_X - \omega_j^2\mathbf{M}_X)\mathbf{H}(\omega_j)_X^N = \mathbf{I} \tag{8}$$

where subscripts A and X denote the FE model and experimental model respectively. $\mathbf{I}$ is the unit matrix, ω_i and ω_j are the one of the circular frequencies for the FE and experimental models respectively.

Premultiplying Eq. (7) by $[\mathbf{H}(\omega_j)_X^N]^T$, one obtains

$$\left[\mathbf{H}(\omega_j)_X^N\right]^T(\mathbf{K}_A - \omega_i^2\mathbf{M}_A)\mathbf{H}(\omega_i)_A^N = \left[\mathbf{H}(\omega_j)_X^N\right]^T\mathbf{I} \tag{9}$$

where the subscript 'T' represents transpose operator.

Premultiplying Eq. (8) by $[\mathbf{H}(\omega_i)_A^N]^T$ yields

$$\left[\mathbf{H}(\omega_i)_A^N\right]^T(\mathbf{K}_X - \omega_j^2\mathbf{M}_X)\mathbf{H}(\omega_j)_X^N = \left[\mathbf{H}(\omega_i)_A^N\right]^T\mathbf{I} \tag{10}$$

Then, transposing both sides of Eq. (9) gives

$$\left[\mathbf{H}(\omega_i)_A^N\right]^T(\mathbf{K}_A - \omega_i^2\mathbf{M}_A)\mathbf{H}(\omega_j)_X^N = \mathbf{I}^T\mathbf{H}(\omega_j)_X^N \tag{11}$$

Subtracting Eq. (11) from Eq. (10), one gets

$$\left[\mathbf{H}(\omega_i)_A^N\right]^T(\mathbf{K}_X - \mathbf{K}_A - \omega_j^2\mathbf{M}_X + \omega_i^2\mathbf{M}_A)\mathbf{H}(\omega_j)_X^N = [\mathbf{H}(\omega_i)_A^N]^T\mathbf{I} - \mathbf{I}^T\mathbf{H}(\omega_j)_X^N \tag{12}$$

It is assumed that the stiffness matrix $\mathbf{K}_X$ of the experimental model is a modification of $\mathbf{K}_A$ to be formulated as

$$\mathbf{K}_X = \mathbf{K}_A + \sum_{n=1}^{N_e}\alpha_n\mathbf{K}_n \tag{13}$$

where $\mathbf{K}_n$ is the stiffness matrix corresponding to the nth element, N_e is the number of elements and α_n are the correction factors to be determined.

Likewise, the corresponding expression for the updated mass matrix $\mathbf{M}_X$ is written as

$$\mathbf{M}_X = \mathbf{M}_A + \sum_{n=1}^{N_e}\beta_n\mathbf{M}_n \tag{14}$$

where $\mathbf{M}_n$ is the stiffness matrix corresponding to the nth element and β_n are the correction factors to be determined.

Substituting Eqs. (13) and (14) into Eq. (12) gives

$$\sum_{n=1}^{Ne}\alpha_n \mathbf{E}_{ij,n} - \sum_{n=1}^{Ne}\beta_n \mathbf{F}_{ij,n} = \mathbf{G} \tag{15}$$

where

$$\mathbf{E}_{ij,n} = \left[\mathbf{H}(\omega_i)_A^N\right]^T \mathbf{K}_n \mathbf{H}(\omega_j)_X^N$$
$$\mathbf{F}_{ij,n} = \omega_j^2 \left[\mathbf{H}(\omega_i)_A^N\right]^T \mathbf{M}_n \mathbf{H}(\omega_j)_X^N$$
$$\mathbf{G} = [\mathbf{H}(\omega_i)_A^N]^T \mathbf{I} - \mathbf{I}^T \mathbf{H}(\omega_j)_X^N + \left[\mathbf{H}(\omega_i)_A^N\right]^T (\omega_j^2 - \omega_i^2)\mathbf{M}_A \mathbf{H}(\omega_j)_X^N$$

Eq. (15) is the updating equation based on normal response function method. Assume the number of the degree of freedom (DOF) of the FE model is s, and the frequency order (the number of ω_i) of the FE model that can be used for updating is x. Similarly, the frequency order (the number of ω_j) of the experimental model that can be used is y and the columns number of every FRF that can be used is n, thus one can obtain $s \times x \times n \times y$ linear equations from Eq. (15). To get the correction factors α_n and β_n by least square solution of Eq. (15), one can gain the updated FE model. When the circular frequencies of the FE model for updating (ω_i) are equal to those of the experimental model (ω_j), Eq. (15) can be simplified into the following form,

$$\sum_{n=1}^{Ne}\alpha_n \left[\mathbf{H}_A^N\right]^T \mathbf{K}_n \mathbf{H}_X^N - \sum_{n=1}^{Ne}\beta_n \omega_j^2 \left[\mathbf{H}_A^N\right]^T \mathbf{M}_n \mathbf{H}_X^N = [\mathbf{H}_A^N]^T \mathbf{I} - \mathbf{I}^T \mathbf{H}_X^N \tag{16}$$

where the symbol '(ω)' respecting their dependence on frequency has been omitted for the sake of brevity.

With the concept of normal FRF, the updating equation is based on the estimate of the normal FRFs which are only affected by mass and stiffness of the structure. Therefore, the proposed improved version of the CMCF method has the ability of updating structural mass and stiffness matrices using complex FRFs for damped system.

2.2. Data incompleteness case

In practice, the DOFs of the FE model greatly exceed those of the experiment model due to time-consuming measurement or sensors insufficiency or measurement location inaccessibility (eg. interface between substructures in a large structural system) or measurement DOFs unavailability (eg. rotational DOF). To solve the problem of the mismatch between the DOFs of the FE model and experimental model, the general adopted approach is model reduction or model expansion. In this note, a different method is proposed for the case of incomplete measurement data.

Assume the circular frequencies of the FE model for updating (ω_i) are equal to those of the experimental model (ω_j) (i.e. $\omega_i = \omega_j = \omega$). Denote the subscripts m and u representing the information of the measured and unmeasured DOFs and denote the subscript r as r th column of FRF matrix. Thus, the governing Eq. (12) can be modified in terms of measured and unmeasured coordinates as

$$\begin{bmatrix} [\mathbf{H}_A^N]_{mm}^T & [\mathbf{H}_A^N]_{mu}^T \\ [\mathbf{H}_A^N]_{um}^T & [\mathbf{H}_A^N]_{uu}^T \end{bmatrix} \begin{bmatrix} \Delta\mathbf{Z}_{mm}^N & \Delta\mathbf{Z}_{mu}^N \\ \Delta\mathbf{Z}_{um}^N & \Delta\mathbf{Z}_{uu}^N \end{bmatrix} \begin{Bmatrix} \mathbf{H}_{Xmr}^N \\ \mathbf{H}_{Xur}^N \end{Bmatrix} = \begin{Bmatrix} [\mathbf{H}_A^N]_{mr}^T \\ [\mathbf{H}_A^N]_{ur}^T \end{Bmatrix} - \begin{Bmatrix} \mathbf{H}_{Xmr}^N \\ \mathbf{H}_{Xur}^N \end{Bmatrix} \tag{17}$$

where $\Delta\mathbf{Z}^N = (\mathbf{K}_X - \mathbf{K}_A - \omega^2(\mathbf{M}_X - \mathbf{M}_A))$.

The first part of Eq. (17) which corresponds to measures DOFs is given below. The remaining part related to unmeasured DOFs is dropped.

$$[\mathbf{H}_A^N]_{mm}^T \Delta\mathbf{Z}_{mm}^N \mathbf{H}_{Xm,r}^N + [\mathbf{H}_A^N]_{mu}^T \Delta\mathbf{Z}_{um}^N \mathbf{H}_{Xm,r}^N + [\mathbf{H}_A^N]_{mm}^T \Delta\mathbf{Z}_{mu}^N \mathbf{H}_{Xu,r}^N + [\mathbf{H}_A^N]_{mu}^T \Delta\mathbf{Z}_{uu}^N \mathbf{H}_{Xu,r}^N = [\mathbf{H}_A^N]_{m,r}^T - \mathbf{H}_{Xm,r}^N \tag{18}$$

where $\Delta\mathbf{Z}_{mm}^N$, $\Delta\mathbf{Z}_{mu}^N$, $\Delta\mathbf{Z}_{um}^N$ and $\Delta\mathbf{Z}_{uu}^N$ are the sub-matrices of the change of initial DSM $\Delta\mathbf{Z}^N$. It is noted that $\mathbf{H}_{Xu,r}^N$ corresponds to DOFs that are unmeasured is replaced by their FE model counterparts $\mathbf{H}_{Au,r}^N$. Due to this approximate treatment, the correction factors of the updating parameters should not be very large, less than 0.5 can be better.

Substituting Eqs. (13) and (14) into Eq. (18) gives the updating equation,

$$\begin{aligned} &\sum_{n=1}^{N_e}\alpha_n([\mathbf{H}_A^N]_{mm}^T [\mathbf{K}_n]_{mm} \mathbf{H}_{Xm,r}^N + [\mathbf{H}_A^N]_{mu}^T [\mathbf{K}_n]_{um} \mathbf{H}_{Xm,r}^N + [\mathbf{H}_A^N]_{mm}^T [\mathbf{K}_n]_{mu} \mathbf{H}_{Au,r}^N + [\mathbf{H}_A^N]_{mu}^T [\mathbf{K}_n]_{uu} \mathbf{H}_{Au,r}^N) \\ &-\omega^2 \sum_{n=1}^{N_e}\beta_n([\mathbf{H}_A^N]_{mm}^T [\mathbf{M}_n]_{mm} \mathbf{H}_{Xm,r}^N + [\mathbf{H}_A^N]_{mu}^T [\mathbf{M}_n]_{um} \mathbf{H}_{Xm,r}^N + [\mathbf{H}_A^N]_{mm}^T [\mathbf{M}_n]_{mu} \mathbf{H}_{Xu,r}^N + [\mathbf{H}_A^N]_{mu}^T [\mathbf{M}_n]_{uu} \mathbf{H}_{Xu,r}^N) \\ &= [\mathbf{H}_A^N]_{m,r}^T - \mathbf{H}_{Xm,r}^N \end{aligned} \tag{19}$$

The correction factors α_n and β_n can be computed just by least square solution.

2.3. Numerical example

To illustrate the validation of the proposed approach, a numerical study of a fixed-fixed beam(30 2D beam elements of length 910 mm, breadth 50 mm and thickness 5 mm) is considered. Each node has two DOFs (translational and rotational). The modulus of elasticity and density are taken as 2.0e+11N/m^2 and 7800kg/m^3 respectively. Proportional structural damping is used, as $0.05\mathbf{K}+0.05\mathbf{M}$. The simulated experimental data is obtained by using a FE model having intentionally introduced certain known discrepancies in the modulus of elasticity of all the finite elements with respect to the FE model. The introduced discrepancies are shown in Table 1, and they are regarded as the correction factors for updating. Three cases are conducted which are the cases of complete data, 50% and 25% incomplete data, respectively. Case of complete data is assumed that both translational and rotational FRFs are available at all the nodes. Cases of 50% and 25% incompleteness are assumed that only translational FRFs are available at all the nodes and at only half of the nodes. Table1 lists the updated correction factors for all the three cases. As it can be seen, all the results are very exact.

TABLE I: The correction factors for different cases

Node number	1	5	7	11	14	18	22	25	27	30
Initial FE model	1	1	1	1	1	1	1	1	1	1
Experimental model	1.2	0.7	0.9	1.1	1.3	0.8	0.7	1.3	1.2	1.1
completeness	1.2	0.7	0.9	1.1	1.3	0.8	0.7	1.3	1.2	1.1
50% incompleteness	1.2	0.7	0.9	1.1	1.3	0.8	0.7	1.3	1.2	1.1
25% incompleteness	1.2	0.697	0.894	1.086	1.33	0.816	0.712	1.274	1.189	1.1

2.4. Conclusions

This paper has outlined an extension to the available CMCF method of model updating to estimate mass and stiffness matrices for damped system. The concept of normal FRF is employed for extracting the FRF of undamped system from the measured complex FRF. Moreover, a simple algorithm for the case of incomplete data is also presented. A numerical example has demonstrated the validation of the proposed method. In the future work, the identification of damping coefficients will be considered.

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