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Failure Load of Plane Steel Frames Using the Yield Surface Method

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Abstract. *In the present work, a nonlinear analysis method of plane steel frames using yield surface method is proposed. The yield function, considering the interaction of bending moment and axial force can be used to determine the elasto-plastic stiffness matrix of beam element used for such structural. The collapse load and the collapse mechanism of plane steel frames are determined by a numerical program in Matlab using the incremental direct method (step by step) and the finite element for which the yield surfaces of Eurocode3 is adopted. Analysis results show that the proposed method is satisfactory.*

Keywords: *nonlinear analysis, steel frame, yields surface, collapse loads, material nonlinear.*

1. Introduction

When the structure is subjected to loads that exceed the proportional limit of the material, the material starts to yield, thus the above mentioned assumptions become inadequate and cannot represent the real behaviour of the structure. In this case plastic analysis is required.

Plastic analysis methods can be classified in two groups: distributed plasticity methods that account for spreading of plastic zones within the whole volume of the structure (Plastic zone methods) and lumped plasticity methods that assume plastic zones to be formed within small area sat the ends of frame members called plastic hinges, while frame members exhibit elastic behaviour between plastic hinges (Plastic hinge methods).

Material yielding, the effects of geometrical non linearities, residual stress and the yield function includes the effect of the stress components acting in the system to predict the yielding of the material are major parameters that control the load-carrying capacity of the structure, and have become a part of many national Standards and Codes (Eurocode3, AISC, British Standards, etc). Moreover, fast-speed personal computers developed in the last 20 years made the use of nonlinear analysis procedures more available for practical purposes.

However, in this case, the discussion will be limited to the material nonlinear, the effects of residual stress and the plasticity is supposed to be concentrated only in the cross section of the ends of the beams (plastic hinge method) and it is in plastic state by the combination of stress that satisfies the yielding condition, interaction of the bending moment with axial force.

Digital program in Matlab has been developed for the load factor calculation by adopting the approach of yield surface. This program uses the finite element method to successive linear analyzes and based on the step by step method.

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2. Yield Surface

The variation of the bending moment with axial force in a cross section can be plotted in terms of the dimensionless quantities N/Np and M/Mp. The resulting curve is called the yield surface because any point on the yield surface represents a state of the fully yielded cross section.

The I or H-shaped sections are often used in steel frames, for which the yield surfaces of Eurocode3 [1] is adopted in present work. The equations are presented bellows:

- Yield surface of Eurocode3 [1] Figure 1 :

$$\frac{M}{0.9Mp} = 1 \qquad \text{for } \frac{N}{Np} \le a/2 \tag{1}$$

$$\frac{M}{0.9Mp} = \frac{1}{1-0.5a}\left(1-\frac{N}{0.8Np}\right) \qquad \text{for } \frac{N}{Np} > a/2 \tag{2}$$

With:

a =min [A_w/A, 0.5]

A: area of section.

A_w: area of web.

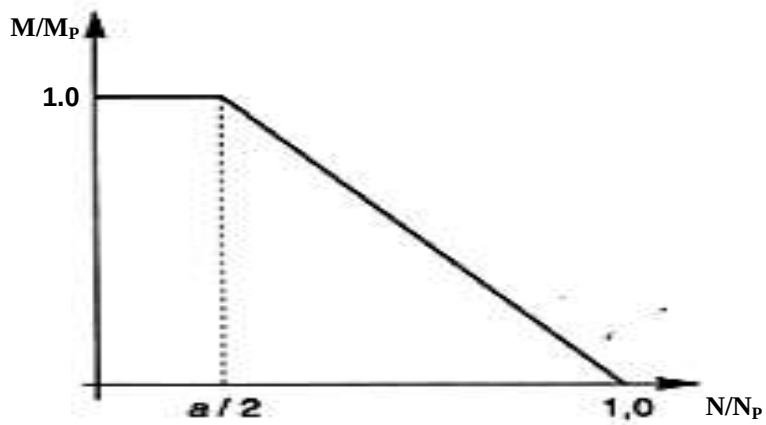

Fig. 1: Yield surfaces of steel I-H sections of Eurocode3

In which, n=N/0.8Np is ratio of the axial force over the squash load, m=M/0.9Mp is the ratios of the major-axis moments to the corresponding plastic moments, and the numbers 0.8 and 0.9 in the denominator account for residual stresses.

2.1. Normality Rule

The yield surfaces described for various cross-sectional shapes can be presented using a yield function Φ such that for a section in a fully yielded state under force interaction Φ =0.

When the effects of bending moment and axial force are taken into account on the yield surface, the associated generalized strains are the rotation and the axial displacement of section. The normality rule was originally proposed by Von Mises in 1928, it may be applied for this case as follows:

$$\{\begin{matrix}\Delta \mathrm{Up} \\ \Delta \theta p\end{matrix}\} = \lambda \left\{ \frac{\partial \Phi}{\partial \mathrm{N}} \frac{\partial \Phi}{\partial \mathrm{M}} \right\}^{\mathrm{t}} \tag{3}$$

Or, symbolically:

$$\{\Delta \mathrm{d_P}\} = \lambda \{f\} \tag{4}$$

Where $\{\Delta \mathrm{d_P}\}$ represent the vector of the plastic deformation increments, λ is the plastic deformation magnitude, $\{f\}$ is a gradient vector at a point of the yield surface Φ.

When the plastic loading occurring, the point force is on the yield surface (or subsequence yield surface) Φ=0. In taking the derivative of this relationship, we obtain:

$$\partial \Phi = \frac{\partial \Phi}{\partial \mathrm{N}} \Delta N + \frac{\partial \Phi}{\partial M} \Delta M = 0 \tag{5}$$

Where the partial derivatives must be taken at the original state of stress resultant. Equation (5) can be written in vectorial form as

$$\Delta \Phi = \{f\}^{\mathrm{t}} \{\Delta \mathrm{P}\} = 0 \tag{6}$$

Where $\{\Delta \mathrm{P}\}$ represent the vector for the increments of stress resultants.

The orthogonal condition can be applied to the relationship between the increments of stress resultants and plastic deformation as implied by Pager's [2] statement that for elastic-perfectly plastic material, "the stress increment does no work on the increment of plastic strain". When applying to frame members, this statement means that

$$\{\Delta \mathrm{dp}\}^{\mathrm{t}} \{\Delta \mathrm{P}\} = 0 \tag{7}$$

For materials in the plastic state, the plastic flow always occurs in association with a dissipation of mechanical energy. Thus, for an increment of plastic deformation $\{\Delta \mathrm{dp}\}$, the dissipative energy ΔW is always positive and is given by:

$$\Delta W = \{\mathrm{P}\}^{\mathrm{t}} \{\Delta \mathrm{dp}\} = \lambda \{\mathrm{P}\}^{\mathrm{t}} \{f\} > 0 \tag{8}$$

2.2. Elastoplastic Stiffness Matrix

For a section in plastic state, the incremental deformation vector, $\{\Delta \mathrm{dp}\}$, consists of both elastic and plastic displacements, depending on which force components are active in the yield function. Hence:

$$\{\Delta \mathrm{d}\} = \{\Delta \mathrm{de}\} + \{\Delta \mathrm{dp}\} \tag{9}$$

Where the incremental elastic displacement vector $\{\Delta \mathrm{de}\}$ is related to the incremental force vector by:

$$\{\Delta \mathrm{P}\} = [\mathrm{K_e}]\{\Delta \mathrm{de}\} = [K_e](\Delta d - \lambda \{f\}) \tag{10}$$

Where $[K_e]$ is the elastic stiffness matrix. Using Equation (10) in Equation (6), the plastic multiplier λ can be found to be:

$$\lambda = \frac{\{f\}^t [K_e]}{\{f\}^t [K_e]\{f\}} \{\Delta d\} \tag{11}$$

Substituting Equation (13) into Equation (12), the elastoplastic stiffness matrix, $\lfloor K_p \rfloor$, can be found:

$$\{\Delta P\} = \lfloor K_p \rfloor \{\Delta d\} \tag{12}$$

Where the elastoplastic stiffness matrix is:

$$\lfloor K_p \rfloor = [K_e] - \frac{[K_e]\{f\}\{f\}^t [K_e]}{\{f\}^t [K_e]\{f\}} \tag{13}$$

Equation (12) is a general expression for a yielded beam element. Since a beam element may be subjected to different combinations of yielding states at its ends, the form of $\lfloor K_p \rfloor$ varies according to the state of yielding and the yield function adopted for plastic analysis.

3. Numerical Verification

3.1. Vogel Portal Frame

The portal frame shown in Figure2 was analysed numerically in 1985 by Vogel [3], and this frame has been used by several researchers (Chen 1993[4], Chen and Kim 1997 [5], Kim and Lu 1992[6], Toma and Chen 1992[7]) as a benchmark solution for including material non-linearities including residual stresses, gradual yielding and full plasticity.

The frame size, material properties and load information are illustrated in Figure 2, and the frame member sizes are listed in Table 1. The horizontal displacement of right upper corner (node A) versus load factor curve by the proposed approach is compared with the plastic zone method and with the plastic hinge method of Vogel (1985) with the plastic hinge method in Figure3. The ultimate load factor obtained by the method proposed is 0.9192 whereas that by Vogel's plastic hinge analysis is λ=1.017 and Vogel's plastic zone analysis is λ=1.02.

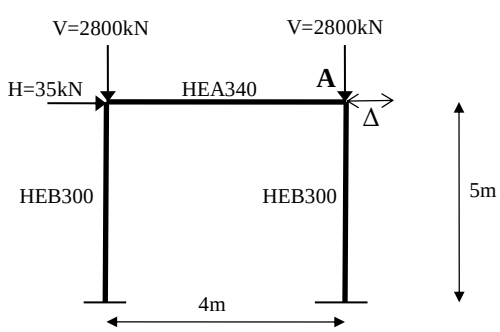

Fig. 2: Geometric configurations and loading pattern of Vogel's frame

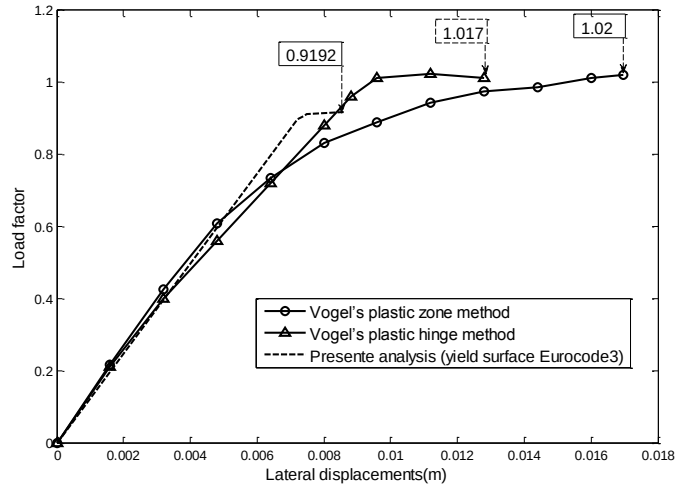

Fig. 3: Load-displacement curve at the top of Vogel's portal frame

TABLE I: Member sizes and sectional properties of the Vogel portal frame

Section properties	h(mm)	b(mm)	t_W	t_f	A(mm^2)	I($x10^6$ mm4)	W_{pl}($x10^3 mm^3$
HEA340	330	300	9.5	16.5	13300	276.9	1850
HEB300	300	300	11.0	19.0	14900	251.7	1869

3.2. Six Storey Frame

The frame size and load information are illustrated in Figure 4 and the frame member sizes are listed in Table 2. The material elastic modulus E of steel is 206kN/mm2 and the yield strength f_y is 235N/mm2. The horizontal displacement of right-upper corner (Node A) versus load factor curve by the yield surface model proposed in this paper is presented in Figure 3. The ultimate load factor λ obtained by the method proposed is 1.2888.

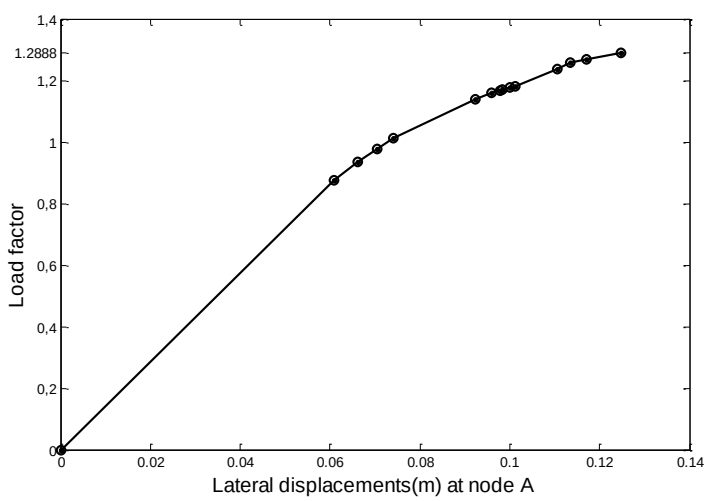

Fig. 3: Load-displacement curve at the top of six storey frame

TABLE II: Member sizes and sectional properties of the six-storey frame

Section properties	h(mm)	b(mm)	t_w(mm)	t_f(mm)	A(mm^2)	I($x10^6$ mm4)	W_{pl}($x10^3 mm^3$
HEA340	330	300	9.5	16.5	13300	276.9	1850
HEB160	160	160	8.0	13.0	5430	24.92	354
HEB200	200	200	9.0	15.0	7810	56.96	643
HEB220	220	220	9.5	16	9100	80.91	827
HEB240	240	240	10.0	17	10600	112.6	1053
HEB260	260	260	10.0	17.5	11800	149.2	1283
HEB300	300	300	11.0	19.0	14900	251.7	1869
IPE240	240	120	6.2	9.8	3910	38.92	367
IPE300	300	150	7.1	10.7	5380	83.56	628
IPE330	330	160	7.5	11.5	6260	117.7	804
IPE360	360	170	8.0	12.7	7270	162.7	1019
IPE400	400	180	8.6	13.5	8450	231.3	1307

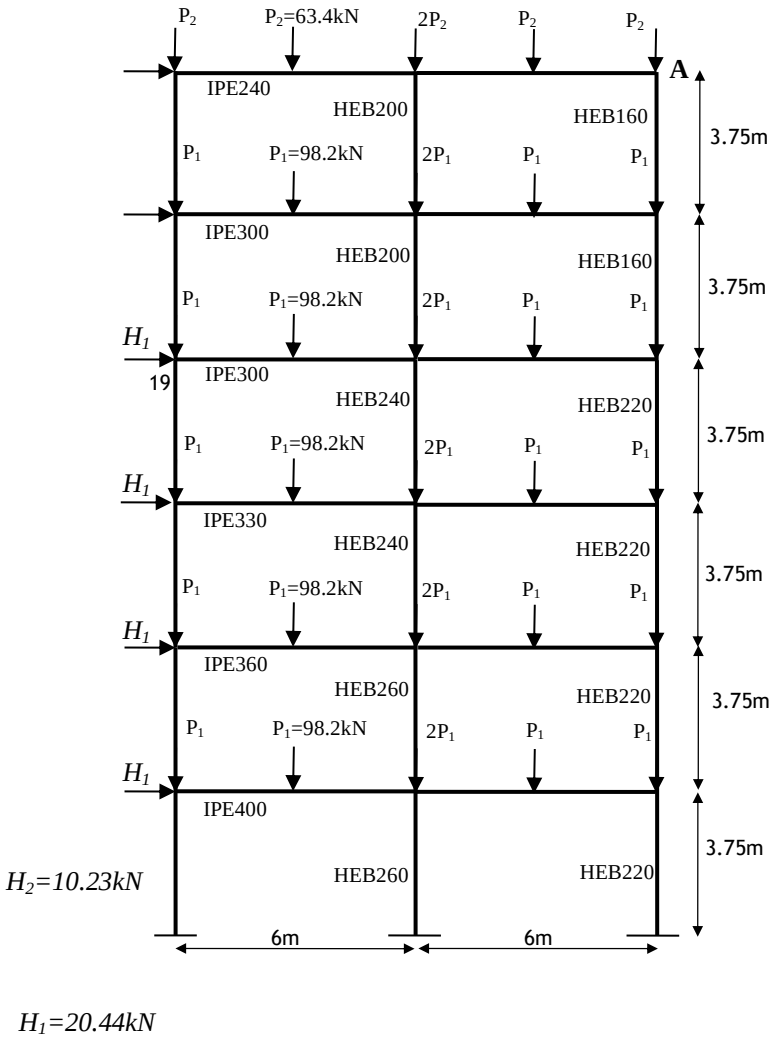

Fig. 4: Six-storey steel frame.

4. Conclusion

An approach for nonlinear analysis of steel frames using yield surface method is proposed in this paper. This approach use the yield function includes the effect of the axial force and bending moment acting in the system to predict the yielding of the material. This approach also considers the influences of the material nonlinear including the residual stress. The numerical results show that the proposed approach is satisfactorily, and is suitable for the nonlinear analysis of steel frames.

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