

## A Bayesian Approach for Structural Health Monitoring of Concrete Bridge

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**Abstract:** Chloride attack from de-icing salt during winter may diffuse through the concrete cover and corrosion will initiate in concrete bridge when the chloride concentration exceed the threshold value. It may lead to loss of strength and unserviceability of the bridge. Structural health monitoring system (SHMS) is used to monitor the corrosion process of reinforcement and has been actively developed recently. However, the monitoring system is subjected to uncertainties associated with material, environmental load and structural

effects. Hence the need for probabilistic analysis expressing life cycle performance in a reliability format. Modelling uncertainty is often associated with limited knowledge which can be reduced by increasing the availability of data. In this study, a method is developed to improve confidence in predicting corrosion concentration with taking into account time dependent reliability analysis. Bayesian updating method is used to update belief by taking into account the prior belief given the likelihood that such event is known. Monte Carlo simulation is used to calculate the probability of failure for annual increment over the life time of the structure based on multiple observations. It is found that by using Bayesian updating, uncertainty of the posterior model is reduced hence increased confidence in predicting future performance of the concrete bridge.

**Keywords:** Corrosion, chloride attack, structural health monitoring, uncertainty, Bayesian updating.

### 1. Introduction

The performance of concrete bridge is reduced by time due to corrosion process especially when the bridge is exposed to aggressive condition such as de-icing salt during winter. In the northern part of United States, the rate of structural deterioration is increasing because of the use of de-icing salts has increased from less than one million tons per year in 1990's [1]. Reinforced concrete subjected to corrosion attack can lead to failure of the structure. In the first stage of corrosion, the corrosion agents such as chloride ions and carbon dioxide will penetrate into the concrete. Secondly, when high concentration of these aggressive agents is achieved, it will break the passive layer of hydrated iron oxide. The passive layer is actually a thin film used to protect the rebar against corrosion. Finally, rust begins to form which is eventually leads to spalling of concrete cover. Chloride attack happened when there is sufficient concentration at the rebar surface to break down the passive layer, chloride ions will act as catalyst to corrosion process which they are not consumed in the reaction but they help to breakdown the passive layer [2]. Basically, monitoring of corrosion process in any types of bridge is essential for the development of well-planned programme to ensure good performance of the bridge during its lifetime.

Structural health monitoring system is used to monitor corrosion process in concrete bridge. The system is developed by installing the corrosion sensor to the bridge to monitor the corrosion process. If the corrosion process can be detected early, deterioration of the bridge can be reduced significantly hence the maintenance cost is reduced. Corrosion monitoring system has been developed by Raupach and Schiessl [3] which permanently monitor the corrosion risk for the reinforcement in concrete structure. These systems enable the owner to take preventive action before damage occurs. Based on electrochemical principles on corrosion process,

Raupach and Schiessl [3] have developed a method to determine penetration of threshold chloride content. Stainless steel or platinum coated titanium act as cathode and reinforced steel electrodes embedded at various depth will act as anode. The installation of sensor of this method can be done on new structure or during repair works on old structure.

The use of probabilistic model to handle the uncertainty present in the deterioration variable is increasing recently. Thoft-Christensen [4] and Sorensen & Engulend [5] are considered to be the first to use a probabilistic framework for corrosion initiation and propagation at rebar level. Then, other researchers such Stewart & Rosowsky [1] and Vu & Stewart [6] have used probabilistic model to predict the bridge performance under chloride attack. Rationally, for decision taken under uncertainty, probabilistic information from all variables influencing the assessment and not just point estimates. Mangat & Molloy [7] have considered diffusion coefficient as a dependent and introduced a different modification. A more thorough approach has been developed by Stewart & Rosowsky [1] to improved probabilistic framework for estimating the time dependent reliability for reinforced concrete decks subject to chloride induced corrosion.

Computing time dependent reliability analysis which is involved the development of probabilistic, behaviour based corrosion initiation and propagation (rate) models. Stewart & Rosowsky [1] have proposed an assessment of reliability of reinforced concrete beam based on computation of reliability index and conclude that the corroded reinforcement has an effect on reliability of exposure condition, quality of concrete and design option. Estes & Frangopol [9] have suggested that reliability analysis need to be updated based on results of inspections to forecast life-cycle performance as reliability methods have gained increasingly with advanced techniques on health monitoring system, the uncertainty in the result can be reduced. However, there has been little discussion about time dependent reliability analysis based on multiple observations.

Bayesian approach has been used by a large number of researchers to update information in order to reduce uncertainty. Faber & Sorenson [10] have used this methodology to update the information regarding the attainment of defined condition states at a given time which is capable of incorporating formally the uncertainty associated with instrument or measurement with updating framework. Bayesian approach also has been used by Rafiq et al. [11] for performance updating of deterioration of concrete bridges fitted with a proactive health monitoring. Thus, this paper addresses a method to improve confidence in predicting corrosion concentration taking into account time dependent reliability analysis using Bayesian framework based on multiple observations.

## **2. Probabilistic Modelling and Simulation**

### **2.1. Deterioration Modelling**

Health monitoring system can be used to identify the governing deterioration mechanism in concrete bridge. Past inspection results is very helpful in selecting the governing mechanism for existing bridges. When the governing mechanism for the particular bridge has been confirm through health monitoring system, the mathematical model can be selected in order to mimic the actual mechanism. This is very important stage because it will be used to predict the deterioration rate at any point in time. In this study, Fick's second law of diffusion is used to predict chloride ion ingress into the concrete bridge. Once the predictive model has been successfully selected, the next step is to update the application. In order to update the governing mechanism, health monitoring system can be used so that any argument between the actual and predicted output can be observed when additional data become available. For this particular study, Bayesian updating approach is used to update variable representing modelling uncertainty thus increasing the confidence by reducing the existing uncertainty in the predictive performance.

### **2.2. Updating Model for Chloride Ingress**

The objective of Bayesian updating procedure is to reduce the uncertainty (i.e. COV) in the predictive performance [11]. In this case, uncertainty in the probability of chloride concentration at given depth and cumulative time exceeds the threshold chloride concentration. Collepardi et al. [12] who first to apply a model used to mimic chloride diffusion in concrete. The first stage in updating procedure is to select the appropriate model to calculate the chloride concentration at given depth and time. Once the chloride concentrations have been calculated, then the probability of failure can be evaluated using the following relation,

$$M(Cxct) = Cth - Cxt \quad (1)$$

$$Pf(Cxct) = P(M \leq 0) \quad (2)$$

Where  $M(Cxct)$  is the margin of chloride concentration and  $Cxt$  is the chloride concentration at cumulative time for which the probability of corrosion is required for decision purposes.  $Cxt$  can be calculated from the following Fick's equation of diffusion

$$C(X_c, t_a) = C_o [1 - erf(X_c \sqrt{2 D.t})] \quad (3)$$

Where  $C_o$  is surface chloride concentration,  $X_c$  is cover depth,  $C_{th}$  is threshold chloride concentration,  $D$  is diffusion Coefficient and  $t$  is time (yearly). The cover depth is set to be 40 mm from the concrete surface. In general, chloride concentration ( $C_{xctaj}$ ) occur within the time interval  $(0, t_L)$  at time  $t_j$  ( $j=1,2,\dots,n$ ) then the cumulative probability of failure of chloride concentration anytime during this time interval is:

$$Pf(t_L) = 1 - \Pr[C_{xcta}(t_1) < C_{cth} \cap C_{xcta}(t_2) < C_{cth} \cap \dots \cap C_{xcta}(t_n) < C_{cth}] \quad (4)$$

Where  $C_{xcta}(t_1)$  represents the initial distribution of chloride concentration and  $C_{xcta}(t_2)$ ,  $C_{xcta}(t_n)$  represent the chloride concentration at time  $t_j$  updated on survival of the previous load events. Technically, the updated chloride concentrations are influenced by time dependent changes in materials properties. Thus the cumulative probability of failure is dependent upon the prior and updated chloride concentration. Monte Carlo simulation with Latin Hypercube Sampling analysis is used as computational procedure.

Updating of bridge reliability can be introduced in two ways which is event updating or distribution updating. For event updating, typically based on single observation and distribution updating is based on multiple observations. For this particular study, distribution updating is adopted herein which was focusing on multiple observations. The simplified form of Bayes Theorem is:

$$\text{Posterior Distribution} = \text{Constant} \times \text{Likelihood} \times \text{Prior Distribution} \quad (5)$$

### 2.3. Simulation of Probabilistic Performance Prediction

In this study, Monte Carlo Simulation with Latin Hypercube Sampling is used to estimate prior and posterior performance prediction of chloride concentration. The output of this simulation is in the form of probability density function of prior, likelihood and posterior distributions. The probability of corrosion initiation for a given time (e.g. 20 years in this case) also presented. The parameters involved in chloride ingress model are summarized below.

TABLE I: Summary of parameters involved in chloride ingress model

Parameter	Mean	C.O.V	Distribution	Reference
$C_o$	3.5 kg/m <sup>3</sup>	0.5	Lognormal	Vu & Stewart (2000)
D (Nominal)	5x10 <sup>-5</sup> m <sup>2</sup> /yr			Vu & Stewart (2000)
Model Error (D)	1.0	0.2	Normal	Vu & Stewart (2000)
			Uniform	
$C_{th}$	0.9 kg/m <sup>3</sup>	0.19	(0.6-1.2 kg/m <sup>3</sup> )	Vu & Stewart (2000)
$X_c$	40 mm	0.1	Normal	Chryssanthopoulos & Sterrit (2002)

## 3. Result and Discussion

### 3.1. Prior Probability of Failure

The evaluation of prior probability of failure is carried out by identifying the frequency of margin of  $C_{th} - C_{xt}$  that falls on or behind the margin value and then divided by total frequency of margin of  $C_{th} - C_{xt}$ . Hence, the prior probability of failure is determined based on previous inspection data. When the new inspection data become available, the updating procedure can be applied by incorporating the new inspection data together with the old data to produce posterior probability of failure. Updating procedure can be performed using Bayesian framework. For the prior probability of failure of chloride concentration, the interval of simulation is one year (e.g. 20 years) at given depth (e.g. 40mm). The cumulative probability of failure is depending upon prior and updated failure margin of chloride concentration. For example, the probability of failure for year three should also consider for probability of failure for the previous year which is year two and one. Fig. 1 shows point in time and cumulative time probability of failure for chloride concentration. The cumulative time probability of

failure gives higher value of probability of failure as compared to the point in time probability of failure. For example, at year 12 the probability of failure for cumulative time is 0.8 and point in time is 0.4 respectively. It is shows that, the probability of failure for cumulative time has increased by about 50% as compared to point in time probability of failure.

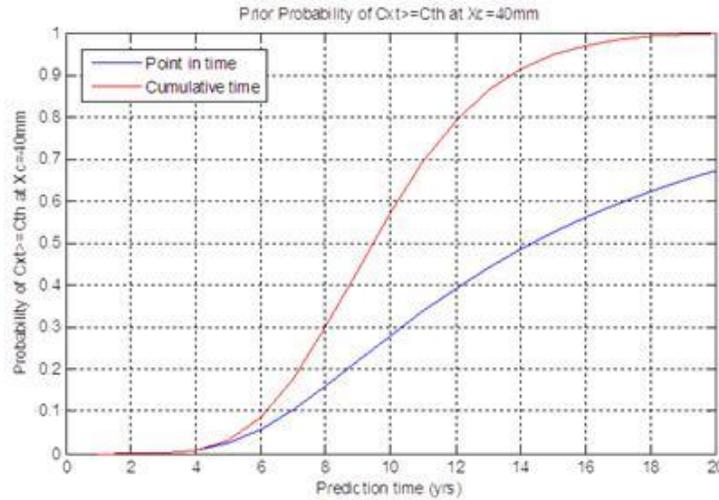


Fig. 1: Point in time and cumulative time probability of failure for chloride concentration

### 3.2. Updating Procedure for Probability of Failure Based on Multiple Observation

Suppose n further independent measurements are made and the likelihood function of  $\theta$  given n independent observations from the normal population  $N(\theta, \sigma^2)$  is

$$l(\theta|y) \propto (1/\sqrt{2\pi\sigma})^n \exp[-0.5(\theta - \theta_o / \sigma_o)^2] \quad (6)$$

The likelihood is

$$l(\theta|y) \propto \exp[-0.5((\theta - \bar{y})/(\sigma/\sqrt{n}))^2] \quad (7)$$

After 10 numbers of observations, the probability density function for chloride concentration has been increased significantly as compared to single observation. This is because the information coming from the data almost completely overrides prior differences. The posterior distribution of chloride concentration is shown in Fig. 2. Based on that figure, it is shows that the variance for posterior distribution is significantly reduced compared to prior and likelihood distribution. Since the variance for posterior distribution is reduced, the uncertainty is also reduced in predictive performance.

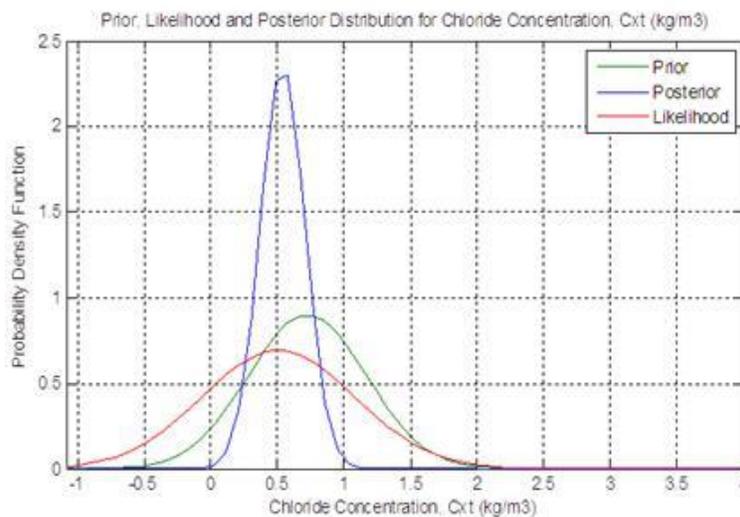


Fig. 2: Prior, likelihood and posterior for multiple observations at year 10

Fig. 3 shows prior and posterior probability of  $C_{xt} \geq C_{th}$  at  $X_c = 40\text{mm}$  for 10 numbers of observations. By increasing the number of observation, the probability of failure for posterior is decreasing significantly as compared to prior. At year 12, the probability of failure for posterior is 0.3 whereas prior is 0.8 respectively. Thus, by increasing the number of observations, the uncertainty can be reduced and increase confidence in predicting the future probability of failure.

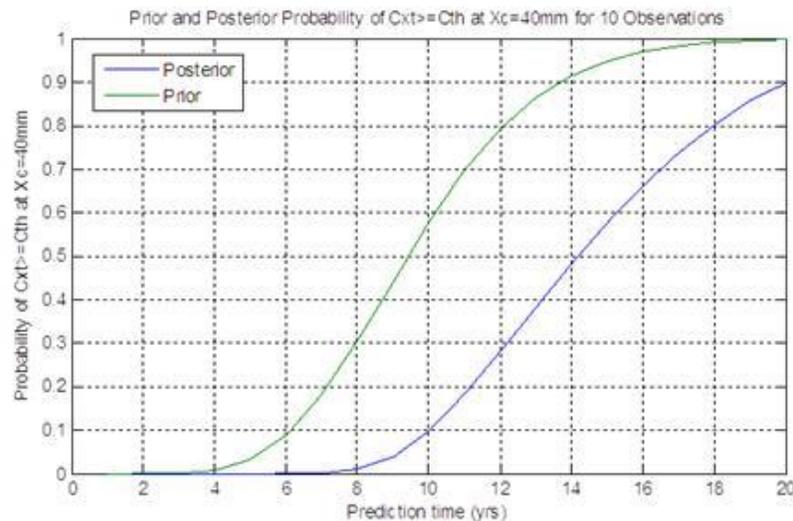


Fig. 3: Prior, posterior probability of failure for multiple observations

## 4. Conclusion

Probabilistic modelling with various parameter defined as variables is presented in this paper. The probabilistic analysis by using Bayesian framework considering time dependent reliability analysis is able to determine the probability of failure. Conceptually, data used to produce likelihood function is obtained from health monitoring system. By health monitoring system installed in a structure and regular inspection, the efficiency of long term performance prediction can be increased. The updating process is carried out with updating based on multiple observations. The result shows that multiple observations yields a tighter probability density function hence reduced probability of failure. It is also found that by using Bayesian updating, uncertainty of the posterior model is reduced hence increased confidence in predicting future performance. Thus, it helps the local authority or the bridge owner to develop a well-planned maintenance program and better allocation of resources.

## 5. Acknowledgement

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